PROTEINS, INTERACTIONS, AND COMPLEXES: A COMPUTATIONAL APPROACH

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Abstract

Proteins play a major role in cellular processes; therefore, it is important to understand how they perform their functions. Proteins, however, do not act alone; they work together to create various biological processes in a hierarchical fashion. First, multiple proteins physically bind together to form stoichiometrically stable complexes. These complexes interact with each other to form functional modules and pathways that carry out most cellular processes. Although large amounts of proteomic data are available, extracting biological insights about proteins and complexes is a challenging task because most high throughput data are noisy and indirect, and thus only weakly correlate with the objective we seek.

In this thesis, we try to understand the hierarchical structure of protein dynamics by applying computational algorithms that deal effectively with the noise and incompleteness in our data. At the lowest level of the hierarchy, we use unsupervised learning to predict the binding sites where the interaction between two proteins occur. At the middle level, we identify a set of stoichiometrically stable complexes. With the availability of labeled data and high quality measurement, we used supervised learning combined with a specifically designed clustering algorithm. Finally, at the highest level, we predict interactions between stoichiometrically stable complexes that belong to the same pathway.

Our methods are validated as more accurate than previous approaches, and they reveal important biological insights. For example, diseases related to certain mutations are shown to involve proteins that are predicted to bind to the sites where the mutations occur, suggesting possible mechanisms where the mutations disrupt the bindings and thus lead to the diseases. Another finding shows that proteins in
larger predicted complexes are more likely to be essential, which explains previous observation that ‘hub’ proteins are more likely to be essential.
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Chapter 1

Introduction

The central dogma of molecular biology states that genetic information is stored in DNA, which is a linear sequence of four nucleotides. When needed, DNA is transcribed into RNA, which in turn is translated into proteins, which are the main cell machinery. The large amount of data produced by many genome projects and ten years of computational analysis of those genomic data provided us with a relatively complete set of genes and their proteins. Analysis of the microarray data produced a picture of when and how much a gene is transcribed, which is a rough estimate of protein abundance. Therefore, the natural next step would be the study of how these proteins perform their functions.

The functions of proteins are as complicated as if not more complicated than DNA or RNA. They work with each other to form various biological processes and pathways in a hierarchical fashion. First, the primary sequence of the protein, which is a linear sequence of 20 amino acids, dictates the folding of the protein into some 3-D structure. Protein properties such as 3-D structure of the protein, the chemical properties of its amino acids, and its localization decide which other proteins or small molecules it physically binds to. Usually the binding happens at places of complementary 3-D structure. This kind of physical association enables multiple proteins to form stoichiometrically stable complexes. At the next level, the complexes interact with individual proteins or other complexes to form functional modules and pathways that carry out most cellular processes. Through this hierarchical structure, the limited
number of proteins are able to combine with each other to perform exponentially
diverse kinds of cellular function.

Recent advances in technology provided us with many types of high throughput
proteomic data, such as yeast two-hybrid and tandem affinity purification for measuring
protein-protein interaction, GFP for measuring protein localization, ChIP-chip
for measuring transcriptional regulation, and double knockout for measuring genetic
interaction. This, combined with high throughput data from DNA and RNA such as
sequence motifs and measurement of mRNA levels of entire genomes under various
conditions, provided us with vast amount of information to understand the protein
interaction and function at different levels of the hierarchical structure.

However, extracting biological insights from these data is a challenging task be-
cause most high throughput data are noisy and many types of data provide indirect
evidence, which only weakly correlate with the biological objective we seek. Fortu-
nately, algorithms in computer sciences, statistics, and machine learning have been
developed to extract patterns from large amount of data while dealing with the above
issues. Therefore, the key to success lies in using the right algorithm among the large
number of possible alternatives, and tailor it to the specific biological data and the
problem we want to solve.

In this thesis, we try to gain understanding of the hierarchical structure of the
protein dynamics by applying a diverse range of computational algorithms, adapted
to the specific problem we want to solve and the characteristics of available data.
At the lowest level, we try to predict binding sites of protein-protein interaction. We
applied the framework of probabilistic graphical model to encode our prior knowledge
about the relationship between different entities. Due to the lack of labeled data and
direct evidence, we used unsupervised learning, which also takes into consideration
the structure of unlabeled parts. At the middle level, we try to predict the protein
composition of stoichiometrically stable complexes. Here we have a reference set of
complexes from small-scale experiments and large amount of direct evidence from
high throughput experiments of relatively high quality. Therefore, we use supervised
learning to combine the evidence and then tackle the complex reconstruction using
a specifically designed clustering algorithm that allows overlap. In the end, at the
highest level, we try to predict interactions between the stoichiometrically stable complexes we just constructed in the previous part. Here again we lack enough labeled data and direct evidence so we used semi-supervised learning. Here we focus on feature construction to extract and aggregate information between two complexes. One useful feature is the protein-protein interactions we predicted in the first part. Therefore, the work of the previous two parts serves as the foundation for the last part, which deals with the highest level of interactions. The common theme across all parts of the thesis is the task of integrating heterogeneous types of noisy data.

1.1 Biological background

Here we go through some basic concepts of molecular biology that are essential in understanding this thesis. We refer the reader to general molecular biology textbook [7] for more information.

Cells are fundamental units of living organisms. The genetic information for making an individual is stored in and replicated through DNA, which is located inside the nucleus. DNA is a sequence of four different types of nucleotides. Certain segments of the DNA correspond to genes, which under appropriate condition would be used to make a molecule called mRNA, whose sequence directly corresponds to the DNA sequence. This process is called transcription. The resulting mRNA migrates out of the nucleus into cell cytoplasm. There, a protein is synthesized in a process called translation using the mRNA as template. A protein is a sequence of 20 different kinds of amino acids. Each amino acid is uniquely determined by three nucleotides on the DNA or RNA. Therefore, once we know the sequence of a gene, we also know the sequence of the corresponding protein.

Different amino acids have different structures and chemical properties. For example, hydrophobicity is how much an amino acid wants to avoid water, i.e. it would be in a high energy (unstable) state when in contact with water molecule. Therefore, a sequence of amino acid in the cell will fold into a specific 3-D structure that minimizes its energy. The hydrophobic, also called non-polar, amino acids will tend to be buried inside while the hydrophilic (polar) ones will be more likely on the surface (Fig. 1.1).
Figure 1.1: A protein is a sequence of amino acids, some of which are hydrophobic (non-polar, green ones) and some of which are hydrophilic (polar, blue ones). It folds into some 3-D configuration in the cell based on the properties of the amino acid with hydrophobic ones buried inside. The 3-D shape of the protein is important to its function.

Amino acids of opposite charge will tend to be close to each other while ones of the same charge are likely to be farther away. The size of the amino acid also puts a constraint on the possible configuration. In general, a protein will adopt certain 3-D structure based on its amino acid sequence although it is difficult to computationally decide its structure based on the sequence.

Proteins do not function in isolation. They physically interact with each other or small molecules (ligands) to mediate biological processes or pathways. The interactions happen when the surface patches of the proteins or ligands complement to each other and form a number of non-covalent bonds such as hydrogen bond, ionic interactions, Van der Waal’s forces, and hydrophobic packing. Therefore, protein structure, esp. its complementarity with other surface patch, plays an important role in facilitating protein-protein interactions. Those contacting surface patches, i.e. protein-protein interaction sites would be an important target when designing a drug to disrupt the interaction.
Computational approaches in predicting the details of protein-protein interactions have not been satisfactory. Docking methods try to find interaction sites by matching two protein structures to find the best sites on both structures [51]. These methods only apply to solved protein structures, which are currently available only for a small number of proteins. We propose an algorithm that identifies protein-protein interaction sites only based on high-throughput data, without explicitly knowing the structures.

Many binary protein-protein interactions come from proteins within the same complex or from proteins between two interacting complexes, where a complex is a stoichiometrically stable set of proteins that permanently associate with each other to play its cellular role as a single unit. For example, the 20S Proteasome complex is consisted of four stacked heptameric ring structures [80] with a total of 28 subunits. The number of unique proteins in the complex varies based on the organism because some subunits share the same protein. The 20S proteasome is the place where proteins are degraded, an important step in many biological processes. In general, complexes are the basic functional units in the cell. Therefore, a faithful reconstruction of the entire set of complexes is essential in understanding the function of individual proteins and the higher level organization of the cell, to which the complexes serve as building blocks.

Fortunately in this case, unlike predicting protein-protein interaction sites described previously or predicting complex-complex interactions described below where we have few labeled data and direct measurement, a new technology called tandem-affinity purification followed by mass spectrometry (TAP-MS) produced large amount of high quality data that measures protein complexes directly [45, 59, 44, 79]. In this assay, a protein, called bait, is fused with a TAP tag. The fusion protein is then introduced into the host and would be able to interact with other proteins under normal physiological conditions. Subsequently, after breaking the cells, the fusion protein is retrieved, together with other constituents attached to it (prey proteins), through affinity selection by means of an IgG matrix. The identity of the bait and prey proteins can be resolved by mass spectrometry. TAP-MS identifies the direct or indirect interaction partners of the bait, which constitute the same complex together.
with the bait. It works under native condition and is able to detect low number of protein copies. The stringent affinity selection method resulted in the identification of mostly stable interactions. Therefore, the assay provides the main signals for our task of predicting stoichiometrically stable complexes.

However, like all high-throughput assays, there are still false positives and negatives in TAP-MS. A tag added to a protein might obscure binding of the bait to its interacting partners. On the other hand, the bait proteins might also retrieve contaminants that is attached non-specifically. Therefore, we might want to consider other data sets, such expression correlation and co-localization, that give signals to as whether two proteins are in the same complex.

The functional roles of protein complexes can be further organized into pathways and processes, where sets of complexes coordinate to achieve a specific goal. For example in a signaling pathway, a protein or complex, which receives signals from upstream entities, interacts with a downstream protein or complex to activate or inhibit its function. Once activated or inhibited, the downstream entity passes the signal further down through more interactions. Therefore, some extra-cellular or environmental perturbation can be amplified into a strong signal in the nucleus. The interactions between upstream and downstream entities usually involves post-translational modification of the downstream entity such as phosphorylation and methylation, which triggers the change of its 3-D configuration and provide energy for its activities. In such cases, the interaction happens only when an upstream signal is received and it ends as soon as the downstream entity is modified. In some other cases, a complex, though an important functional unit, is not able to perform a biological role by itself. Instead, it needs to assemble with other complexes into a bigger body. For example, One copy of 20S proteasome assembles with two copies of 19S proteasomes into a 26S proteasome which performs the protein degradation where the 19S proteasome regulates the entry of the proteins into 20S proteasome, where the protein is destructed. In this example, the 26S proteasome is assembled only when needed and the assembly requires the binding of ATP to the 19S ATP-binding sites. In general, complexes in the same pathway interact with each other to coordinate the execution of certain biological processes. These interactions tend to be transient. They happen in specific
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Once the process is done, their association may disappear.

Understanding the behavior of different biological pathways leads us to the ultimate goal of biology — predicting the phenotype. On the way from DNA (genotype) to phenotype, there are a lot of other important biology we need to understand such as the number of mRNA copies that are produced and phosphorylation, methylation, and other post-translational modification of the proteins. However we refer the readers to the textbooks since those contents are not directly related to this thesis. Here we focus on the part that goes from the underlying mechanism of interaction between two proteins, to the interactions between two complexes, with the main theme around protein complexes.

1.2 Overview of the thesis

Following is an overview of the rest of the chapters in this thesis:

Chapter 2: Protein-protein interaction sites: Protein-protein interactions happen at specific places on the protein sequences. A mutation occurring inside the interaction site can disrupt the particular protein-protein interaction and thus leads to some disease. Drugs has been designed to specifically target the interaction sites in order to disrupt harmful protein-protein interactions. In this chapter, we predict interactions between proteins, as well as the location of the interaction sites. Our method takes the following input:

1. protein motifs, which are conserved patterns on protein sequences that recur in many proteins. There are many existing motif databases that are derived from high throughput sequence data. Longer motifs are sometimes called domains, which is usually a functional unit.

2. evidence for protein-protein interactions, such as yeast two-hybrid or TAP-MS, and indirect evidence like co-expression.

3. evidence for motif-motif interactions such as domain fusion.
The output are predicted interaction probability for a pair of proteins and the confidence that the interaction occurs at a specific site.

We use a probabilistic graphical model, Bayesian networks [107], to encode the relationships between the inputs and outputs. Probabilistic models are a powerful framework that provides a principled integration of heterogeneous types of data and deals with noise effectively. One challenge is that few known interaction sites are available as true labels, especially outside the model organism of Saccharomyces cerevisiae; there are no high-throughput assays and individual experiments using co-crystallization are costly and time-consuming [13]. Therefore in this unsupervised setting, instead of training the Bayesian network discriminatively, we trained it generatively by maximizing the likelihood of the observed data while summing over the missing labels. Such likelihood function, however, is non-convex and direct optimization is difficult. We solve this problem by applying Expectation Maximization algorithm (EM), which is guaranteed to find a local optimum.

Our predictions on protein-protein interactions and interaction sites are shown to have better accuracy than other state-of-the-art methods in terms of correctly predicting reliable protein-protein interactions and the interaction sites from co-crystallized data in PDB. Diseases related to certain mutations are shown to involve proteins that are predicted to bind to the sites where the mutations occur, suggesting possible mechanisms where the mutations disrupt the bindings and thus lead to the diseases.

Chapter 3. MRF for protein-protein interactions and complexes: Many of the protein-protein interactions we observe in the previous chapter are derived from proteins in the same complex: if protein $A$ interacts with $B$ and $B$ interacts with $C$, it is likely that $A$, $B$, and $C$ are in the same complex and thus $A$ also interacts with $C$. This transitivity relationship suggests that instead of predicting the interaction between each pair of proteins independently, we can try to predict all of them ‘collectively’ at the same time by exploiting the correlation among them; we can also take into account relationships that involve
other types of data such as if $A$ transcriptionally regulates both $B$ and $C$, then $B$ and $C$ are more likely to interact. We demonstrate how to do this to improve the accuracy on protein-protein interactions in the first half of this chapter.

The task of ‘collective classification’, where a set of labels are predicted together while considering their dependencies, fits well into the framework of Markov Random Fields (MRF). The MRF, like the Bayesian Network, is a kind of probabilistic graphical model. It is a powerful framework and a principle way to encode prior domain knowledge about the relationships between different entities. It allows us to collectively predict all the unknown variables while taking into consideration the correlation between those predictions, such as the transitivity relationship. There are vast amount of research devoted to the efficient learning and inference of probabilistic graphical models in general, and MRF in particular. However, most approaches are still too slow or only approximate, which severely limit the application of MRF. Therefore, we extended one class of inference algorithm, which is fast and exact but limited to a special class of MRF. The new algorithm, while still being fast, can be applied to a wide range of MRF, including ones that represent interesting problems in biology. We applied the model to the problems of predicting all interactions between proteins. We demonstrate the significant speedup of the new algorithm and show the collective predictions are more accurate than a flat model where each prediction is made independently based on its own features.

The transitivity relationship we use is largely a result of multiple proteins associating with each other to form a complex. So why not predict the complex directly? With the recent availability of large amount of high quality measurement of co-complexed proteins, it becomes possible for a genome-wide reconstruction of complexes. MRF, which is a flexible framework, can be readily applied to construct a model for this task. In the second half of this chapter, we apply the above fast inference algorithm to the new MRF for the task of predicting protein complexes.
Chapter 4. Stoichiometrically stable complexes: The previous approach of using an MRF for predicting complexes has low coverage. In this chapter, we construct a comprehensive set of stoichiometrically stable complexes in Saccharomyces cerevisiae. The goal here is to improve the accuracy by integrating heterogeneous types of data and train the model carefully so as to predict at the level of protein complexes, instead of functional modules.

We use supervised learning for this problem because there are large amounts of direct measurements and enough labeled training data derived from a reference set of complexes. Here our choices are over which algorithm to use and what features to construct. In the case of MRF, the likelihood of a set of proteins being a complex depends on the sum of the affinities for all pairs of proteins within the set. This limits the possible types of features we can construct. Therefore, we tried alternative methods which create a rich set of features directly from the multiple types of evidence between all pairs of proteins, instead of first combining them into pairwise affinities protein pair by protein pair. Classification algorithms such as Boosting, logistic regression, and Support Vector Machine (SVM) are based on a flat model where each prediction is made independently; this limitation is offset by the rich features these methods can incorporate and the fast and powerful learning methods. We tried different algorithms on the problem. The winner turns out to be a two-stage approach combining LogitBoost, a variant of Boosting, and an extension of hierarchical agglomerative clustering (HAC) that allows overlap (HACO). LogitBoost is first used to predict co-complex likelihood (affinity) between two proteins from multiple types of evidence; then HACO is used to cluster the resulting pairwise affinity graph. This approach worked the best because LogitBoost is able to select important and complementary features automatically from large amount of heterogeneous biological data. The list of features selected helps us understand the relationship among and relative strength of the many types of evidence.

Our set of predicted complexes is shown to be more accurate and biologically more coherent than the predictions from other state-of-the-art methods. We
are able to identify novel complexes, which are consistent with other sources of evidence. Finally, our predicted set of complexes allows us to better understand the essentiality of the genes. Previous studies have found the relationship between essentiality and the degree of the protein in the protein-protein interaction network. We show, however, that the size of the complex to which the protein belongs is a better predictor of the protein’s essentiality than its degree.

Chapter 5. interactions between complexes: A pathway usually involves a set of stoichiometrically stable complexes that work together to achieve a specific biological task. In the process, complexes interact with each other to coordinate their activities for different purposes.

Interaction brings two complexes physically close to each other so they can work together on some substrate. In some cases, one complex processes the substrate to produce some intermediary, and the other complex processes the intermediary to produce the final product; by interacting and being in physical proximity, the two-step process can be completed efficiently. In other cases, a bigger body needs to be assembled from several complexes, which play related roles to achieve a task.

Interaction also brings closer two complexes so one complex modifies the other, such as phosphorylation and methylation. The modification either activates or inhibits the other complex by altering its 3-D configuration and providing it with energy.

These interactions, however, happen only when they are needed for the specific biological task, such as in response to the change in the environment. Therefore, they are more transient in nature, as they occur only under specific condition, and at specific time and location. In this chapter, we predict interactions between the set of high quality complexes we constructed in the previous chapter.

There are few known complex-complex interactions because their transient nature makes experimental detection difficult. On the other hand, computational studies on interactions between complexes are limited by the lack of a comprehensive set of known complexes. To address the lack of labeled data, i.e. known
complex-complex interactions, we apply a Naive Bayes model with hidden variables for unknown interaction status and train it generatively using EM. Most signals for complex-complex interactions are defined over protein pairs, while our prediction task is between two complexes. Therefore, we aggregate the signals between these two multi-protein complexes to construct rich features that are used to predict the interactions between these two complexes.

Using cross-validation, we show that the interactions we predict have high accuracy. They are enriched for complexes in the same pathway or functional categories. We annotate each pair of interactions with the transcription factors that regulate them and the condition in which they are activated. This helps biologists to understand the specific condition, time, and location the interaction happens and what biological processes and pathways it is involved in.

We also applied the same model to the protein-complex interactions. With the high-quality protein-protein interaction predictions from Chapter 2, we produced a unified interaction network involving both proteins and complexes.

Chapter 6. Conclusions and future directions: We summarize this thesis by talking about its contribution and limitations. We also discuss challenges and future directions.

1.3 Our contribution

In this thesis, we provide a machine learning framework that can be applied to a wide range of problems related to the hierarchical organization of proteins into high level entities. Its flexibility makes it possible to integrate in heterogeneous types of data and deals with noise effectively, which are the two main challenges given the large amount but noisy data in the field of proteomics. Here is a list of our specific contributions:

Biological:

1. High quality and genome-wide predictions of protein-protein interactions and their binding sites.
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2. A set of reference complexes that is merged from different sources with higher coverage.


4. A better way to process time-series expression data. Among many ways to process the data, this correlates the best with interactions between complexes.

5. High quality and genome-wide predictions of interactions between complexes and proteins.

All the above predictions can be downloaded from our website for further analysis by biologists.

Computational:

1. An algorithm that allows us to do fast MAP inference in MRF.

2. An extension to the popular hierarchical agglomerative clustering (HAC) algorithm to allow overlaps (HACO) in the resulting clustering. Since HAC is shown to be useful in many tasks [34], we expect HACO to be also widely applicable.

All the above novel algorithms as well as the code that generated our biological predictions can be downloaded from our website. They are general-purpose and can be applied to a wide range of problems.
Chapter 2

Protein-protein interaction sites

In this chapter, we propose InSite, a computational method that integrates high-throughput protein and sequence data to predict protein-protein interactions and infer the specific binding regions of interacting protein pairs. We compared our predictions with binding sites in Protein Data Bank and found significantly more binding events occur at sites we predicted. Several regions containing disease-causing mutations or cancer polymorphisms in human are predicted to be binding for protein pairs related to the disease, which suggests novel mechanistic hypotheses for several diseases.

2.1 Introduction

Much recent work focuses on generating proteome-wide protein-protein interaction maps for both model organisms and human, using high-throughput biological assays such as affinity purification [45, 59, 44, 79] and yeast two-hybrid [123, 115, 48, 132, 127, 63]. However, even the highest-quality interaction map does not directly reveal the mechanism by which two proteins interact. Interactions between proteins arise from physical binding between small regions on the surface of the proteins [21]. By understanding the sites at which binding takes place, we can obtain insights into the mechanism by which different proteins fulfill their role. In particular, when mutations alter amino acids in binding sites they can disrupt the interactions, often changing the behavior of the corresponding pathway and leading to a change in phenotype. This
mechanism has been associated with several human diseases [68]. Thus, a detailed understanding of the binding sites at which an interaction takes place can provide both scientific insight into the causes of human disease and a starting point for drug and protein design.

We propose an automated method, called InSite (for Interaction Site), for predicting the specific regions where protein-protein interactions take place. InSite assumes no knowledge of the 3-D protein structure, nor of the sites at which binding occurs. It takes as input a library of conserved sequence motifs [39, 38], a heterogeneous data set of protein-protein interactions, obtained from multiple assays [44, 79, 127, 63, 96, 134], and any available indirect evidence on protein-protein interactions and motif-motif interactions, such as expression correlation, Gene Ontology (GO) annotation [9], and domain fusion. It integrates these data sets in a principled way and generates predictions in the form of ‘motif M on protein A binds to protein B’.

InSite is based on several key assumptions. The first is that protein-protein interactions are induced by interactions between pairs of high-affinity sites on the protein sequences. Second, we assume that most binding sites are covered and characterized by motifs or domains. (For simplicity, we use the word ‘motif’ to refer to both motifs and domains, except in cases where we wish to refer specifically to domains.) Although an approximation, this assumption is supported in the literature, as interaction sites tend to be more conserved than the rest of the protein surface [19]. These motifs can correspond to any conserved pattern recurring on protein sequences, whether short regions or entire domains. Finally, we assume that the same motifs participate in mediating multiple interactions. Therefore, we can study a motif’s binding affinity with other motifs by examining multiple protein-protein interactions that involve the motif.

InSite is structured in two phases. In the first phase, the algorithm searches for a set of affinity parameters between pairs of motif types that provides a good explanation of the interaction data, roughly speaking: (a) every pair of interacting proteins contains a high-affinity motif pair, (b) non-interacting proteins do not contain such motif pairs, and (c) motif pairs with supporting evidence such as from domain fusion should be more likely to have high affinity. There may be multiple assignments
to the affinity parameters that explain the data well; our method tends to select
sparser explanations, where fewer motif pairs have high affinity, thereby incorporating
a natural bias towards simplicity. A simple example of this phase is illustrated in
Fig. 2.1; here, the observed interactions are best explained via high affinity for the
motif pair \( a,d \), explaining the interactions \( P_1 - P_3 \) and \( P_1 - P_4 \), and high affinity
for the pair \( b,e \), explaining the interactions \( P_1 - P_3 \) and \( P_2 - P_5 \). By contrast, the
motif pair \( c,d \) is not as good an explanation, because the motif pair also appears
in the non-interacting protein pair \( P_3, P_5 \). We note that the motif pair \( a,c \) is also a
candidate hypothesis, as it predicts the interactions \( P_1 - P_3 \) and \( P_1 - P_5 \) and does not
incorrectly predict any other interaction. However, it leaves the interaction \( P_1 - P_4 \)
unexplained, therefore leading to a less parsimonious model that also contains the
motif pair \( a,d \).

A set of estimated affinities provides us with a way of predicting, for each pair of
proteins, which motif pair is most likely to have produced the binding. In the second
phase, we use this ability to produce specific hypotheses of the form ‘Motif \( M \) on
protein \( A \) binds to protein \( B \)’. In a naive approach, we can simply take the most
likely set of binding sites for the estimated set of affinity parameters. However, in some
cases, there may be multiple models that are equally consistent with our observed
interaction pattern, but that give rise to different binding predictions. In the second
phase of InSite, we therefore assess the confidence in each binding prediction by
‘disallowing’ the \( A - B \) binding at the predicted motif \( M \), re-estimating the affinities,
and computing the overall score of the resulting model (its ability to explain the
observed interactions). The reduction in score relative to our original model is an
estimate of our confidence in the prediction. This phase serves two purposes: it
increases the robustness of our predictions to noise, and also reduces the confidence
in cases where there is an alternative explanation of the interaction using a different
motif. For example, in Fig. 2.1, the prediction that ‘motif \( d \) on \( P_4 \) binds to \( P_1 \)’
has higher confidence, because \( d \) is the only motif that can explain the interaction.
Conversely, the prediction that ‘motif \( d \) on \( P_3 \) binds to \( P_1 \)’ has lower confidence,
because the motif pair \( a,c \) can provide an alternative explanation to the interaction.
The prediction that ‘motif \( e \) on \( P_5 \) binds to \( P_2 \)’ also has high confidence: although
Figure 2.1: Example illustrating the intuition behind our approach. In this simple example, there are five proteins (elongated rectangles) with four interactions between them (black lines); proteins contain occurrences of sequence motifs (colored small elements within the protein rectangles). Pairs of motifs on two proteins may bind to each other and hence mediate a protein-protein interaction if they have high affinity. The observed interactions are best explained via high affinity for the motif pair \(a, d\), explaining the interactions \(P_1 - P_3\) and \(P_1 - P_4\), and high affinity for the pair \(b, e\), explaining the interactions \(P_1 - P_5\) and \(P_2 - P_5\). We can now estimate the confidence in a prediction ‘\(P_i\) binds to \(P_j\) at motif \(M\)’ by (computationally) ‘disabling’ the ability of \(M\) to mediate this interaction. For example, the prediction that \(P_1 - P_4\) bind at motif \(d\) has high confidence, because \(d\) is the only motif that can explain the interaction. Conversely, the prediction that \(P_1 - P_3\) bind at motif \(d\) has lower confidence, because the motif pair \(a, c\) can provide an alternative explanation to the interaction. The prediction that \(P_2 - P_5\) bind at motif \(e\) also has high confidence: although interaction via binding at \(b, c\) would explain the interaction, making \(b, c\) a high-affinity motif pair would contradict the fact that \(P_2\) and \(P_5\) do not interact.
interaction via binding at $b, c$ would explain the interaction, making $b, c$ a high-affinity motif pair would contradict the fact that $P_2$ and $P_3$ do not interact.

We provide a formal foundation for this type of intuitive argument within an automated procedure (Fig. 2.2), based on the principled framework of probability theory and Bayesian networks [107]. At a high level, the InSite model contains three components, which are trained together to optimize a single likelihood objective. The first component, inspired by the work of Deng et al. [31] and Riley et al. [112], formalizes the binding model described above, whereby motif pairs have binding affinities, and an interaction between two protein pairs is induced by binding at some pair of motifs in their sequence. The second and third components, novel to our approach, formulate the evidence models for protein-protein interactions and motif-motif interactions respectively. They address both the noise in high-throughput assays [83, 130], and in the case of protein-protein interactions, the fact that many of the relevant assays are based on affinity purification, which detects protein complexes instead of the pairwise physical interactions that are the basis for inferring direct binding sites. To integrate many assays coherently, InSite uses a naive Bayes model [83, 100, 65], where the assays are a ‘noisy observation’ of an underlying ‘true interaction’.

Our entire model is trained using the expectation maximization (EM) algorithm in a unified way (see Section 2.4 and Fig. 2.5), to maximize the overall probability of the observed protein-protein interactions. This type of training differs significantly from most previous methods that aggregate multiple assays to produce a unified estimate of protein-protein interactions. These methods [65, 143] generally train the parameters of the unified model using only a small set of ‘gold positives’, typically obtained from the MIPS database [96]. This form of training has the disadvantages of training the parameters on a relatively small set of interactions, and also of potentially biasing the learned parameters towards the type of interactions that were tested in small-scale experiments. By contrast, the use of the EM algorithm allows us to train the model using all of the protein interactions in any data set, increasing the amount of available data by orders of magnitude, and reducing the potential for bias. The same EM algorithm also trains the affinity parameters for the different motif pairs, so as to best explain the observed protein-protein interactions.
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Figure 2.2: **Overview of our automated procedure.** Our automated procedure (InSite), which has two main phases, takes as input protein sequences and multiple evidences on protein-protein interactions and motif-motif interactions.

(a) Motifs, downloaded from Prosite or Pfam database, were generated based on conservation in protein sequences. Protein-protein interactions are obtained from a variety of assays, including: a small set of ‘reliable’ interactions, which recurred in multiple experiments or were verified in low-throughput experiments; a set of interactions from yeast two-hybrid assays; and a set of interactions from the co-affinity precipitation assays of Krogan et al. [79] and Gavin et al. [44].

(b) The first phase (Fig. 2.4 and Fig. 2.5) uses a Bayesian network to estimate both the motif pair binding affinities and the parameters governing the evidence models of protein-protein interactions and motif-motif interactions, where the model is trained to maximize the likelihood of the input data. Note that the affinity learned in this phase only depends on the type of motifs, regardless of which protein pair they occur on.

(c) In the second phase (Fig. 2.6), we do a protein-specific binding site prediction based on the model learned in the previous phase. For each protein pair, we compute the confidence score for a motif to be the binding site between them. Note that the confidence scores computed here are protein specific and can be different for the same motif depending on the context it appears in.
These estimated affinities allow us to predict, for each pair of proteins, which motif pair is most likely to have produced the binding. In the second phase, we use these predictions, augmented with a procedure aimed at estimating the confidence in each such prediction, to produce specific hypotheses of the form ‘Motif M on protein A binds to protein B’. In this phase, InSite modifies the model so as to enforce that binding between A and B does not occur at motif M. We then compute the loss in the likelihood of the data, and use it as our estimate of the confidence in the binding hypothesis.

As an initial validation of the InSite method, we first show that it provides high-quality predictions of direct physical binding for held-out protein interactions that were not used in training. These integrated predictions, which utilize both binding sites and multiple types of protein-protein interaction data, provide high precision and higher coverage than previous methods. As the primary validation of our approach, we compare the specific binding site predictions made by InSite to the co-crystallized protein pairs in the Protein Data Bank (PDB) [13], whose structures are solved and thus binding sites can be inferred. In our results, 90.0% of the top 50 Pfam-A domains that are predicted to be binding sites are indeed verified by PDB structures. InSite significantly out-performs several state-of-the-art methods: In particular, only 82.0% of the top 50 predictions by Lee et al. [82] and 80.0% of the top 50 predictions by Riley et al. [112] and of Guimaraes et al. [53] are verified in PDB.

We also examined the functional ramifications of our predictions. If protein A interacts with protein B via the motif M on A, a mutation at motif M may have a significant effect on the interaction. If the interaction is critical in some pathway, this mutation may result in a deleterious phenotype, which may lead to disease [119]. We applied InSite to human protein-protein interaction data, and considered those predicted binding motifs M that contain a mutation in the OMIM human disease database [54] or identified as a potential driver mutation in the recent cancer polymorphism data [52]. We then investigated the hypothesis that the mutation at M leads to the disease by disrupting the binding of the protein pair. A literature search validated many of these disease-related predictions, whereas others are unknown but provide plausible hypotheses. Therefore, our predictions provide us with significant
insights into the underlying mechanism of the disease processes, which may help future study and drug design.

We have made our predictions and our code publicly available for download [1]. Our algorithm is general, and can be applied to any organism, any protein-protein interaction data set, and any type of motifs or domains.

2.2 Related work

Deng et al. [31] constructed a Bayesian Network that tries to best explain the observed protein-protein interactions by motif-motif interactions. Their simple Bayesian Network, however, does not take into account indirect evidence. Instead, it only uses motifs and observed protein-protein interactions, with the goal of better predicting the interactions, not the interaction sites. Liu et al. [88] used the same Bayesian Network but incorporated protein-protein interactions from three organisms to gain better accuracy at predicting protein-protein interactions. Gomez et al. [50] used a model, in which a motif pair can be repulsive — reducing the interaction probability of a protein pair containing the motif pair. Again, their goal is to use the protein sequence information to help better predict protein-protein interactions.

Our approach is most similar to previous work that tries to predict motif-motif or domain-domain interactions [53, 82, 112, 102]. A key difference between InSite and previous methods is that InSite makes predictions at the level of individual protein pairs, in a way that takes into consideration the various alternatives for explaining the binding between this particular protein pair. By contrast, other methods predict affinities between motif types; these predictions are independent of the proteins on which the motifs occur. For example, Guimaraes et al. tries to explain protein-protein interactions using as fewer motif-motif interactions as possible. They formulate the problem using linear programming where the variables to be solved are potential interactions between two motif types. Lee et al. proposed a new measure, the expected number of interactions between two motif types, and used a Bayesian approach to integrate it with information on motif pairs such as domain fusion and GO similarity.

Whereas the above methods aim to compute the general affinity between two motif
types, InSite also explicitly computes the confidence that a specific motif occurrence mediates the binding of a specific interacting protein pair. It may give the same motif pair different binding confidences in the context of explaining different protein-protein interactions. These finer-grained predictions allow us to identify the specific mechanism for their interaction, whereas other methods that make predictions by only looking at motif types would not be as appropriate for this purpose. For example, the DPEA method by Riley et al. [112] also uses a Bayesian Network that tries to best explain protein-protein interactions by motif-motif interactions. Besides some of the algorithmic problems, which we will discuss in the Methods section, it treats all observed protein-protein interactions as gold positives and thus neglects the noises in those assays. No indirect evidence is integrated either for protein-protein interactions or for motif-motif interactions.

Most importantly, DPEA computes the confidence score between a pair of motif types by forcing them to have affinity 0. In contrast, InSite aims to compute predictions for a specific motif occurrence on an interacting protein pair, and thus forces a particular motif occurrence on a particular protein to be non-binding to another protein. The more global perturbation used by Riley et al. would not be as appropriate for this purpose: It may well be the case that a good alternative binding hypothesis exists for the interaction at a particular protein pair, but disallowing all interactions between a pair of motif types causes significant reduction to the likelihood in other protein pairs. Indeed, our method outperforms DPEA, and other state-of-the-art methods like the parsimony approach by Guimaraes et al. and the integrative approach by Lee et al., at identifying binding regions between an interacting protein pair. To our knowledge, InSite is the first method that does protein specific binding site predictions. This capability allows us to use InSite to understand specific disease-causing mechanisms that may arise from a mutation that disrupts a protein-protein interaction.

Some other work [103, 67] infers motif-motif interaction using other types of information. Jothi et al. [67] observed that interacting domain pairs for a given interaction exhibit higher level of co-evolution than the non-interacting domain pairs. Motivated by this finding, they developed a computational method to test the generality of
the observed trend, and to predict large-scale domain-domain interactions. Given a protein-protein interaction, their method predicts the domain pairs that are most likely to mediate the interaction. They applied the method to yeast and its predictions has been shown to have little overlap with InSite-style methods [67], and thus can be combined with InSite to gain wider coverage.

InSite also provides a unified framework for integrating evidence from multiple assays, some of which are noisy and some of which are indirect. Unlike other methods, our approach uses all available evidence for both protein-protein interactions and motif-motif interactions, and it does not assume the existence of a large data set of gold positives.

2.3 Sources of data

We extracted signals from multiple sources of data and integrated them using our Bayesian Network model.

2.3.1 S. cerevisiae

We constructed a ‘gold standard’ set of S. cerevisiae protein-protein interactions from MIPS [96] and DIP [134], downloaded on March 21st, 2006. We extracted from MIPS those physical interactions that are non-high-throughput yeast two-hybrid or affinity chromatography. For DIP, we picked non-genetic interactions that are derived from small-scale experiments or verified by multiple experiments. We use this set of reliable interactions as ‘gold standard’ interactions in our model. For ‘gold standard’ non-interactions, we picked 20,000 random pairs [12] and removed those that appear in any interaction assays. For these gold standard pairs, we fixed the value of the ‘actual interaction’ variable accordingly. In all other protein pairs, we leave the actual interaction variables as unobserved.

We constructed ‘observed interaction’ variables for each of the assays, as follows. For the yeast two-hybrid data sets of Uetz et al. [127] and Ito et al. [63], these variables are binary-valued. They take the value true if the pair is observed to interact in the
assay, and the value false if both of the two proteins appeared in the assay but the pair was not observed to interact. However, as the number of unobserved interactions grows quadratically in the number of proteins assayed, this procedure would result in too many non-interacting pairs; we therefore keep only those pairs that appeared in some other high-throughput data set, to allow evidence integration. For the TAP-MS assays, we selected the interactions with confidence score above 0.2 from Krogan et al. [79] and all interactions from Gavin et al. [44], using their confidence scores as continuous observation values.

This procedure results in a data set of 101,065 protein pairs, of which 4,200 were gold standard interactions and 18,666 gold standard non-interactions, and a total of 108,924 observations. See Fig. 2.3.

We computed expression correlation using a compendium of time series data obtained in different environmental conditions [139, 95, 20, 81, 106, 43, 42, 32, 70]. The compendium has 76 different conditions with a total of 403 time points. For each pair of proteins, we computed the Pearson correlation coefficient across all the time points. We also annotated our proteins with biological process from GO. For each pair of proteins, we computed the GO distance as the log size of the smallest common category shared by the two proteins. The smaller the value, the more specific category the two proteins belong to, and thus they are more likely to interact [111].

In one run, we used sequence motifs from the Prosite database [38] excluding the non-specific motifs, mostly post-translational modification motifs that appears across many proteins. We removed motifs that are annotated as ‘Compositionally biased’ or ‘DNA or RNA associated’. This gives us 708 different types of motifs with a total of 2,808 motif occurrences. In another run, we used sequence motifs from the Pfam domain database [11], which results in 8,089 different types of domains with a total of 11,767 domain occurrences.

We construct a ‘domain fusion’ variable for each pair of Prosite motifs or Pfam domains. Its value is 1 if the two motifs ever co-occur on the same protein in any species whose proteins are sequenced and annotated in the motif databases. Its value is 0 otherwise. Note that we use the term ‘domain fusion’ here although it can also refer to motifs. We also looked at whether the two motifs appear together in any
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Figure 2.3: Protein-protein interaction assays. A total of 101,065 pairs are used, among which 4,200 are reliable interactions and 18,666 are gold non-interactions (see Methods). Each of the remaining pairs is associated with observations from one or more of the four high-throughput assays. The size of the four circles represents the number of pairs in each of the experimental assays. The red slice within each circle represents the overlap with reliable interactions, while the blue slice represents the overlap with gold non-interactions. In Gavin’s and Krogan’s assays, each pair is associated with a confidence score. In Ito’s and Uetz’s assays, we have either observed interacting pairs (3,938 for Ito and 821 for Uetz) or non-interacting pairs, which are between the proteins used in the assay but not identified as interacting. The number on the line between two circles is the number of pairs that overlap between the two assays, with only positive interactions considered in the case of the Ito and Uetz assays. Following is a breakdown of the number of pairs in each assay:

<table>
<thead>
<tr>
<th>Assay</th>
<th>Gold PPI</th>
<th>Other observed PPI</th>
<th>Gold non-PPI</th>
<th>Other observed non-PPI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gavin</td>
<td>1157</td>
<td>69,140</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>Krogan</td>
<td>847</td>
<td>6,591</td>
<td>N.A.</td>
<td>N.A.</td>
</tr>
<tr>
<td>Ito</td>
<td>1,542</td>
<td>2,396</td>
<td>7,362</td>
<td>16,343</td>
</tr>
<tr>
<td>Uetz</td>
<td>599</td>
<td>222</td>
<td>706</td>
<td>2,019</td>
</tr>
</tbody>
</table>

**Observed non-interactions**
**Reliable interactions**
**Observed interactions**
**Actual non-interactions**

*x.x:* # overlaps between two sets of interactions
biological process category based on the mapping table from Pfam to GO [9]. If they do, we assign the ‘shared GO’ variable to be 1 and we assign it to be 0 otherwise.

2.3.2 Human

We used a high confidence yeast two-hybrid assay [115] and the Human Protein Reference Database (HPRD), a resource that contains known protein-protein interactions manually curated from the literature by expert biologists [108] (downloaded on Jan. 24th, 2006). The union of these data sets gives us 6,688 reliable interactions. We also used a yeast two-hybrid assay from Stelzl et al. [123] and an assay that identify co-complex proteins [37] with its confidence score as our observation value. This gives us 5,723 observations. As in yeast, we picked 20,000 random pairs as our gold non-interactions [12] and removed those that appear in any interaction assays. We used the same Prosite motifs, which gives us 687 different types of motifs with a total of 3,034 motif occurrences.

2.4 Methods

2.4.1 Probabilistic model

Our probabilistic model has three components. The first (Fig. 2.4, black box) formalizes the binding model described above: for each protein pair in our model, and each pair of motifs on the two proteins, we have a variable indicating whether binding took place at this motif pair. The prior probability that a specific motif pair binds is the affinity of the corresponding motif types. The overall interaction of the proteins is a disjunction of these binding events, and of an additional ‘spurious binding’ variable that accounts both for noise in some interaction data sets and for binding outside of motifs in our database. The second component of our model (Fig. 2.4, red box) addresses the problem that very few protein interactions are known with certainty. Yeast two-hybrid assays can be noisy [83, 130], with a non-trivial fraction of both false positives and false negatives, while affinity purification detects protein complexes instead of the pairwise physical interactions that are the basis for inferring
direct binding sites. Moreover, indirect evidence such as co-expression, though useful, only weakly correlates with the actual interactions. Therefore, to integrate many assays coherently, we use a naive Bayes model [83, 100, 65]. In this model, we have an ‘Interaction variable’ for each protein pair, whose value is ‘true’ only when the pair actually interacts. This variable is unobserved in most cases, but serves to aggregate information from a set of partial and noisy assays, which are viewed as ‘noisy sensors’ for the interaction variable. The quantitative dependencies of these sensors are modeled differently for different assays, to allow for variations in false positive and false negative rate [130, 86], and for confidence scores accompanying certain assays [44, 79].

The parametric families of the dependency relationships are picked by examining the data and their parameters are fitted when the model is learned. There may be multiple observation variables attached to a protein pair, whose interaction probability summarizes the signal from all the assays and is used to learn the binding affinity. The third component of our model (Fig. 2.4, blue box) takes into consideration the noisy evidence on motif-motif interactions. A binding variable between two motifs may have multiple evidences, all of which serve as noisy sensors for the binding variable and are integrated using a naive Bayes model in the same way as in the second component. Note parameters of the evidence models for motif-motif interactions are all also learned from the data. Some of the learned values are illustrated in Fig. 2.4.

More formally, each interacting or non-interacting pair of proteins $P_i, P_j$ is described by an entity $T_{ij}$. A pair of motifs in two proteins can potentially bind and induce an interaction between the corresponding proteins. We encode this assumption by introducing a variable $T_{ij}.B_{ab}$ for each pair of motifs $a$ in $P_i$ and $b$ in $P_j$, which represents whether the pair of motif occurrences actually binds. The probability that they bind depends on the affinity between the motifs. Therefore, we define:

$$P(T_{ij}.B_{ab} = true) = \theta_{ab}$$

and

$$P(T_{ij}.B_{ab} = false) = 1 - \theta_{ab}$$
Figure 2.4: Our Bayesian Network model. The first phase of InSite, shown here, uses a Bayesian network to estimate the affinities between motif pairs and evidence models for protein-protein interactions and motif-motif interactions. The Bayesian network is trained so as to maximize the likelihood of the observed protein-protein interaction and motif-motif interaction pattern. An illustrative fragment of the Bayesian network, for the protein pair $P_1 - P_3$ of Fig. 2.1, is shown inside the box. The variable $B$ represents the actual binding of a protein pair at a particular motif pair, which is never observed, but is shown to correlate with information such as domain fusion and Gene Ontology. We use them as noisy indicators ($E$), which take binary values — whether two motifs ever appear in the same protein and whether they share the same GO biological process category. The variable $S$ represents ‘spurious’ binding, which occurs at a region not represented in our set of motif pairs. An actual interaction between the proteins, represented by the variable $I$, occurs whenever any type of binding occurs. Importantly, not all of the data represent high-reliability physical binding between protein pairs: some data sets could be noisy, and the affinity precipitation assays capture entire complexes. We therefore assign the variable $I$ the value true in the training data only if the protein pair is a high-reliability physical interaction and assign the value false if it is among the randomly picked 20,000 pairs. If the pair occurs in high-throughput assays or has some indirect evidence, it is treated as a noisy indicator ($O$). For the binary interaction assays, this indicator is a binary-valued variable; the protein-complex assays of Gavin and Krogan are associated with a numerical score, and are treated as continuous-valued indicators, whose parametric form was derived by examining the data. Indirect evidence such as expression correlation and Gene Ontology, which is shown to correlate with protein-protein interaction, are also used as noisy indicators. Arrows in the Bayesian network represent the variable downstream is probabilistically dependent on its parent variable. The observed variables are colored in orange and stripes are used for partially observed variables. Both the motif binding affinities and the parameters governing the evidence models are learned together using the expectation maximization algorithm, to maximize the likelihood of the observed data. Some examples of the evidence models learned are shown in the call-out boxes.
where $\theta_{ab}$ is the affinity between motifs $a$ and $b$. Note that this affinity is a feature of the motif pair and does not depend on the proteins in which they appear. We place a Dirichlet prior distribution over the value of $\theta_{ab}$, which is the same for $\theta$ across all motif pairs. We must also account for interactions that are not explained by our set of motifs, such as the binding between amino acids not included in our motif set. Thus, we add a *spurious binding* variable $T_{ij}.S$. The probability that spurious binding occurs is given by:

$$P(T_{ij}.S = \text{true}) = \theta_s(m) = 1 - (1 - \theta_s)^m$$

where $m$ is proportional to the average (geometrical) number of amino acids not covered by any motif in the two proteins. It represents the fact that the more amino acids we have outside the motif set, the more likely the interaction is induced by something other than binding between motifs. Two proteins interact if and only if some form of binding occurs, whether by a motif pair or by spurious binding. Thus, we define a variable $T_{ij}.I$, which represents whether protein $P_i$ interacts with protein $P_j$, to be a deterministic OR of all the binding variables $T_{ij}.S$ and $T_{ij}.B_{ab}$. We note that Riley *et al.* [112] did not include a spurious interaction variable in their model, but rather used 0.001, regardless of the protein length, as the probability of interaction when there is no motif pair between two proteins.

To account for the fact that our experimental assays are not direct and reliable measurements of physical protein-protein interactions, we define the observation variables $T_{ij}.O$ to be the interactions observed in the experimental assays and indirect evidence like co-expression and GO distance, which are noisy sensors for the actual interaction variable $T_{ij}.I$. Note that an actual interaction variable may have several observation variables if the pair appears in multiple assays. For those assays with binary observations, $T_{ij}.O_n$ is a binary variable and the probability it is true depends on $T_{ij}.I$ and the type of assay. Therefore, we can account for the different false positive and false negative rates in different assays. For Gavin *et al.* [44], we assume her confidence score $T_{ij}.O_g$ to be Gaussian distributions, whose mean and variance depends on the $T_{ij}.I$. For Krogan *et al.*, we assume the confidence score $T_{ij}.O_k$ has a uniform
distribution if $T_{ij}.I$ is \textit{false} (non-interacting) and has an exponential distribution if $T_{ij}.I$ is \textit{true} (interacting). For co-expression, we assume the Pearson correlation coefficient $T_{ij}.O_e$ to have a Gaussian distribution, whose mean and variance depends on the $T_{ij}.I$. For GO distance, we assume its value $T_{ij}.O_o$ to be an exponential distribution when $T_{ij}.I$ is \textit{false} and a mixture of Gaussian and uniform distribution when $T_{ij}.I$ is \textit{true} (interacting). In the case of human confidence score $T_{ij}.O_w$ from Ewing \textit{et al.}, we use a mixture of Gaussian and indicator functions with different parameters depending on the value of $T_{ij}.I$.

We use $R_{ab}$ to describe a pair of motif $a$ and motif $b$. We introduce a variable $R_{ab}.E_g$ to represent whether they share the same GO biological process category and another variable $R_{ab}.E_f$ for whether they appear together in a domain fusion event. Both variables are probabilistically dependent on the binding variable $T_{ij}.B_{ab}$ and serve as its noisy sensors. Note that $R_{ab}$ is the same regardless which protein pair $T_{ij}$ it appears in. We use different models for domain fusion and GO distance to account for their different correlation with the actual motif-motif interactions.

An instantiation of our probabilistic model is illustrated in Fig. 2.4 and the conditional probabilities involved are summarized below:
\[ P(\theta_{ab} = x) = P(\theta_s = x) \]
\[ = \frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1 - x)^{\beta-1} \]

\[ P(T_{ij} \cdot B_{ab} = true | \theta_{ab} = x) = x \]

\[ P(T_{ij} \cdot S = true | \theta_s = x) = \theta_s(m) \]
\[ = 1 - (1 - x)^m \]

\[ T_{ij} \cdot I = OR(T_{ij} \cdot B_{ij} \cdot S) \]

\[ P(T_{ij} \cdot O_n | T_{ij} \cdot I) = \rho_n(T_{ij} \cdot I) \]

\[ P(T_{ij} \cdot O_g | T_{ij} \cdot I = false) = N(\mu_{g0}, \sigma_{g0}^2) \]

\[ P(T_{ij} \cdot O_g | T_{ij} \cdot I = true) = N(\mu_{g1}, \sigma_{g1}^2) \]
\[ P(T_{ij} \cdot O_k | T_{ij} \cdot I = false) = 1 \]

\[ P(T_{ij} \cdot O_k | T_{ij} \cdot I = true) = \lambda_k \exp(-\lambda_k (1 - T_{ij} \cdot O_k)) \]

\[ P(T_{ij} \cdot O_e | T_{ij} \cdot I = false) = N(\mu_{e0}, \sigma_{e0}^2) \]

\[ P(T_{ij} \cdot O_e | T_{ij} \cdot I = true) = N(\mu_{e1}, \sigma_{e1}^2) \]
\[ P(T_{ij} \cdot O_o | T_{ij} \cdot I = false) = \lambda_0 \exp(-\lambda_0 (8.68 - T_{ij} \cdot O_o)) \]

\[ P(T_{ij} \cdot O_o | T_{ij} \cdot I = true) = w_{o1} N(\mu_{o1}, \sigma_{o1}^2) + w_{o2} U(7, 8.68) \]

\[ P(T_{ij} \cdot O_w | T_{ij} \cdot I = false) = w_{w1} N(\mu_{w0}, \sigma_{w0}^2) + w_{w2} I(T_{ij} \cdot O_w = 0) \]
\[ + w_{w3} I(T_{ij} \cdot O_w = NA) \]

\[ P(T_{ij} \cdot O_w | T_{ij} \cdot I = true) = w_{w4} N(\mu_{w1}, \sigma_{w1}^2) + w_{w5} I(T_{ij} \cdot O_w = 0) \]
\[ + w_{w6} I(T_{ij} \cdot O_w = NA) \]

\[ P(T_{ij} \cdot O | T_{ij} \cdot I) = \prod_{T_{ij} \cdot O \in T_{ij} \cdot O} P(T_{ij} \cdot O | T_{ij} \cdot I) \]

\[ P(R_{ab} \cdot E_g | T_{ij} \cdot B_{ab}) = \rho_g(T_{ij} \cdot B_{ab}) \]

\[ P(R_{ab} \cdot E_f | T_{ij} \cdot B_{ab}) = \rho_f(T_{ij} \cdot B_{ab}) \]

\[ P(T_{ij} \cdot E | T_{ij} \cdot B_{ab}) = P(R_{ab} \cdot E_g | T_{ij} \cdot B_{ab}) P(R_{ab} \cdot E_f | T_{ij} \cdot B_{ab}) \]

where \( \alpha, \beta \) are the hyper-parameters in the Dirichlet distribution, which encodes our
prior belief about the evidence model before seeing any data. 8.68 is the maximum value of the GO distance, $\rho$ represents binary functions. $n$ enumerates the different type of yeast two-hybrid assays, $O_g$ is Gavin et al.’s assay, $O_k$ is Krogan et al.’s assay, $O_c$ is co-expression, $O_o$ is GO distance, $O_w$ is Ewing’s confidence score, $E_g$ is shared GO motif function, $E_f$ is domain fusion, $U()$ is uniform distribution, and $I()$ is indicator function. The observation parameter vector $\eta$ is the union of $\rho$, $\mu$, $\sigma$, $w$, $\lambda$.

2.4.2 Learning

The model defines a joint probability over the entire set of attributes, which is the product of all local conditional probability models shown above. Our learning objective is to find affinities between motifs $\theta$, probability of spurious binding $\theta_s$, and the parameters for the observation models $\eta$, which maximize the probability over observed evidence on protein-protein interactions $T.O$, the partial assignment to the actual interactions $T.I$, and the observed evidence on motif-motif interactions $R.E$. Our algorithm uses an iterative procedure based on the Expectation-Maximization (EM) algorithm [30] to find the local maximum. In the E-step, we compute the conditional probabilities for the binding variables $T.B$, $T.S$, and the actual interaction variables $T.I$, given $\theta$, $\theta_s$, $\eta$, $T.O$, $R.E$, and use those as the soft assignments to the variables. Define:

$$P(T_{ij}.B_{ab} = true|R_{ab}.E) = \theta'_{ab} \frac{\theta_{ab}P(R_{ab}.E|T_{ij}.B_{ab} = true)}{P(R_{ab}.E)}$$

To be the binding probability given the evidences on motif-motif interactions, where:

$$P(R_{ab}.E) = P(T_{ij}.B_{ab} = true)P(R_{ab}.E|T_{ij}.B_{ab} = true) + (1 - P(T_{ij}.B_{ab} = true))P(R_{ab}.E|T_{ij}.B_{ab} = false)$$

By Bayes’ rule, we have:
\[ P(T_{ij} \cdot B_{ab} = true | T, O; \theta, \eta) = \frac{\theta'_a b P(T_{ij}, O | T_{ij}, I = true)}{P(T_{ij}, O)} \]

\[ P(T_{ij} \cdot S = true | T, O; \theta, \eta) = \frac{\theta_s(m) P(T_{ij}, O | T_{ij}, I = true)}{P(T_{ij}, O = true)} \]

\[ P(T_{ij} \cdot I = true | T, O; \theta, \eta) = \frac{P(T_{ij}, I = true) P(T_{ij}, O | T_{ij}, I = true)}{P(T_{ij}, O)} \]

where

\[ P(T_{ij}, I = true) = 1 - (1 - \theta_s(m)) \prod_{T_{ij}, B_{ab} \in T_{ij}, B} (1 - \theta'_a b) \]

\[ P(T_{ij}, O) = P(T_{ij}, I = true) P(T_{ij}, O | T_{ij}, I = true) \]

\[ + (1 - P(T_{ij}, I = true)) P(T_{ij}, O | T_{ij}, I = false) \]

In the M-step, we compute relevant expected sufficient statistics using the computed soft marginal probabilities as soft assignments. We use maximum a posteriori (MAP) inference to re-estimate the parameters \( \theta, \theta_s, \eta \). This step can be executed efficiently in closed form, using standard methods, for the parameters \( \theta, \eta \). To estimate \( \theta_s \), we need to decompose it into \( m \) variables and apply EM to this approximate form. In detail, the probability that a spurious binding occurs between a protein pair is given by:

\[ P(T_{ij} \cdot S = true) = \theta_s(T_{ij}, m) = 1 - (1 - \theta_s) T_{ij} \cdot m \]

where \( T_{ij}, m \) is proportional to the average (geometrical) number of amino acids not covered by any motif in the two proteins. This is equivalent to having \( m \) variables \( T_{ij} \cdot s_k, k = 1, \ldots, m \), each of which has probability \( \theta_s \) of being true, and \( T_{ij} \cdot S \) is the deterministic OR of these \( m \) variables.

To learn \( \theta_s \), in E-step, we compute the posterior marginal probabilities for \( T_{ij} \cdot s_k \):
EM Learning algorithm

Observed: O, E, partial I
Hidden: B, S, partial I

Repeat:

Estimation of:
• motif affinities (θ)
• noisy observation models (η)
  • protein-protein interactions
  • domain-domain interactions

E step → M step

Soft assign to:
• B, S, I

Figure 2.5: Schematic illustration of our EM-based learning algorithm. It estimates the motif affinities (θ) and parameters in the evidence model (η) based on the InSite model illustrated in Fig. 2.4.
\[ P(\text{\(T_{ij}\cdot s_k = \text{true}\)}|\text{T.O}; \theta, \eta) = \frac{\theta_s P(\text{T_{ij.O}|T_{ij.I = \text{true}}})}{P(\text{T_{ij.O = \text{true}}})} \]

In M-step, we re-estimate \( \theta_s \) by:

\[ \theta_s = \frac{\sum_{i,j} \sum_{k=1}^{T_{ij.m}} P(\text{T_{ij.s_k = true}|T.O}; \theta, \eta)}{\sum_{i,j} T_{ij.m}} \]

Note that we would not have a closed-form solution if we do not decompose \( T_{ij.S} \) into \( T_{ij.s_k} \) and instead try to re-estimate \( \theta_s \) directly from:

\[ P(\text{T_{ij.S = \text{true}|T.O}; \theta, \eta) = \frac{\theta_s(T_{ij.m})P(\text{T_{ij.O}|T_{ij.I = \text{true}}})}{P(\text{T_{ij.O = \text{true}}})} \]

We repeat the E-step and M-step until the change of likelihood is less than a threshold. Since, in the next phase, we force each motif-protein pair to be non-binding and compare the change of likelihood \( L_{iaj} \), we have to make sure the threshold used here for convergence is at least a magnitude smaller than \( L_{iaj} \), so the noise would not overwhelm the signal. Here we set the threshold to be 0.01 in terms of change of log-likelihood. Note that DPEA of Riley et al. [112] used the change in expected log-likelihood to test for convergence. This does not optimize the joint likelihood, which may not always increase over the EM steps. On the other hand, InSite used joint likelihood, which is the measure we try to optimize and is guaranteed to increase after each EM iteration.

To estimate the two hyper-parameters, \( \alpha, \beta \) of the Dirichlet distribution, we used two-fold cross-validation on the PDB data set. In this regime, we select the hyper-parameters so as to optimize performance on one PDB fold, and evaluate performance on the other fold; thus, no data in the test set was used to estimate any of the parameters or hyper-parameters in the model.

\subsection*{2.4.3 Binding confidence estimation}

Since we explicitly model the binding events between a pair of motifs and between amino acid pairs outside the motif set, it gives us a way to compute the confidence that
a motif on a protein binds to another protein. Here the intuition is that if a motif is non-binding for a particular interaction, it is dispensable from the model. We first run our model until convergence. To predict whether motif \( a \) on protein \( i \) is the binding site to protein \( j \), we force \( a \) not to bind with any motif on protein \( j \) (Fig. 2.6). We rerun our algorithm with the above constraint and use the change in likelihood as the confidence score of our prediction, which we denote to be \( L_{iaj} \). A high score indicates that forcing \( a \) not to be the binding site for proteins \( i \) and \( j \) induces a big change in likelihood and is unfavorable. A low score suggests the binding site is dispensable from the model with competing hypotheses that can explain the observed interactions, and thus the prediction is questionable. Unlike the motif affinities \( \theta_{ab} \) learned from the previous step, here our confidence score \( L_{iaj} \) depends on both proteins \( i \) and \( j \) and is different for different proteins.

Note Riley \textit{et al.} force a pair of motif types to have affinity 0 and thus, unlike us, their prediction is not specific to each individual protein pair. Also, their change of likelihood is computed only based on the interacting protein pairs while throwing away information that can be gained from non-interacting pairs. In contrast, InSite uses the likelihood of the entire model, which forces us to explain both interactions and non-interactions.

### 2.4.4 Model initialization

If a motif pair does not appear between any pair of interacting proteins, we set its affinity to be 0, an assignment guaranteed to maximize the joint likelihood; this helps simplify our model structure. We set the initial affinity for the remaining motif pairs based on the frequency they appear between interacting protein pairs [121]. The observation parameters \( \eta \) for the evidence models are initialized based on empirical counts for the ‘gold standard’ interactions and non-interactions.
Figure 2.6: **Perturbation analysis for binding site prediction.** In the second phase of InSite, we do protein-specific binding site prediction based on the model we learned in the previous phase (see Fig. 2.4 and Fig. 2.5). For each protein pair, we compute the confidence score for a motif to be the binding site between them. This procedure, illustrated here, estimates the effect, on the model likelihood, of disallowing binding at the predicted motif. For example, to estimate the confidence in the prediction that the $P_1 - P_3$ binding takes place at motif $a$, we remove the binding variables ($B$) for motif pairs $(a,c)$ and $(a,d)$ and their associated noisy indicators ($E$), thereby preventing $a$ from being used for binding. We use the change in likelihood as the confidence in this prediction.
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2.5 Results

2.5.1 Overview

We applied InSite to data from both S. cerevisiae and human. For S. cerevisiae, we compiled 4,200 reliable protein-protein interactions as our gold standard and 108,924 observations of pairwise protein-protein interactions from high-throughput yeast two-hybrid assays of Ito and Uetz [63, 127] and assays of Gavin and Krogan that identify complexes [44, 79]. We also computed expression correlation and GO distance between every pair of proteins, data which have been shown to be useful in predicting protein-protein interactions [111]. Altogether, these measurements involve 4,669 proteins and 82,399 protein pairs. We also constructed a set of fairly reliable non-interactions as our gold standard by selecting 20,000 random protein pairs [12], and eliminating those pairs that appeared in any interaction assay. In the case of human, we used two sets of training data for our analysis. First, we focused on high-confidence pairwise interactions, all of which were modeled as gold positive interactions. These interactions were obtained both from high-quality yeast two-hybrid assays [115] and from the Human Protein Reference Database (HPRD), a resource that contains published protein-protein interactions, manually curated from the literature [108]. In the second case, we additionally incorporated into our evidence model the yeast two-hybrid interactions from Stelzl et al. [123] and the assay from Ewing et al. [37] that identifies complexes. Overall, we obtained 12,411 protein interactions involving 2,926 proteins, and selected 18,745 random pairs as our gold non-interactions, as for yeast.

The InSite method can be applied to any set of sequence motifs. Different sets offer different trade-offs in terms of coverage of binding sites; we can estimate this coverage by comparing residues covered by a particular set of motifs to residues found to be binding sites in some interaction in PDB. One option is Prosite motifs [38], where we excluded non-specific motifs, such as those involved in post-translational modification, which are short and match many proteins. These motifs cover 9.6% of all residues in the protein sequences in our data set (Fig. 2.7(a)). Of residues that are found to be binding sites in PDB, 37.8% are covered by these Prosite motifs. This enrichment is significant, but many actual binding motifs are omitted in this analysis.
Figure 2.7: Motif coverage of protein sequences and binding sites. Motif (domain) coverage of protein sequences compared with coverage of the protein-protein interaction binding sites in yeast. The outer circle represents all residues in all 4,669 protein sequences we used in our data set and the light blue is the portion that is covered by our motifs (domains). The inner circle represents residues that are identified to be binding sites in PDB. It only includes the 268 proteins in our data set that are crystallized in PDB and thus whose binding sites we can infer. The dark blue is the portion that is covered by our motifs (domains).

(a) Prosite motifs. 9.6% of the residues in our data set are covered by Prosite motifs, 37.8% of the binding residues in PDB-included proteins are covered by Prosite.

(b) Pfam domains. 73.9% of the residues in our data set are covered by Pfam domains, 70.9% of the binding residues in PDB-included proteins are covered by Pfam.

(c) Pfam-A domains. 38.1% of the residues in our data set are covered by Pfam-A domains, 70.3% of the binding residues in PDB-included proteins are covered by Pfam-A.
An alternative option is to use Pfam domains [11], which cover 73.9% of all the residues; however, PDB binding sites are not enriched in Pfam (Fig. 2.7(b)). Pfam-A domains (Fig. 2.7(c)), which are accurate, human crafted multiple alignments, appear to provide a better compromise: Pfam-A domains contain only 38.1% of the residues in our dataset, but cover 70.3% of the PDB binding sites. One regime that seems to work best, which is also used by Riley et al., is to train on all Pfam domains (providing a larger training set) and to evaluate the predictions only on the more reliable Pfam-A domains. For each motif set, we used evidence from domain fusion and whether two motifs share common GO category as noisy indicators for motif-motif interactions [35, 92].

We experimented with different data sets and different motif sets. In each case, we trained our algorithm on these data; then, for each interacting protein pair, we compute the binding confidences for all their motifs, and generate a set of binding site predictions, which we rank in order of the computed confidence.

2.5.2 Predicting physical interactions

The actual protein-protein interactions are mostly unobserved in our probabilistic model. However, we can compute the probability of interaction between two proteins based on our learned model, which integrates evidences on protein-protein interactions and motif-motif interactions as well as the motif composition of the proteins. As a preliminary validation, we first evaluate if InSite is able to identify direct physical interactions. We compare our results to those obtained by using the confidence scores computed by Gavin and Krogan, which are derived from their TAP-MS assays and quantify the propensity of proteins to be in the same complex. Using standard ten-fold cross-validation, we divide our gold interactions and high-throughput interactions into ten sets; for each of ten trials, we hide one set and train on the remaining nine sets together with our gold non-interactions. We then compute the probability of physical interaction for each protein pair in the hidden set, and rank them according to their predicted interaction probabilities. We define a predicted interaction to be true only if it appears in our gold interactions, and false if it appears only in the
high-throughput interactions; we then count the number of true and false predictions in the top pairs, for different thresholds. Although this evaluation may miss some true physical interactions that appear in the high-throughput data set but not in our gold set, it provides an unbiased estimate of our ability to identify direct physical interactions. We separately perform this procedure by ranking the interactions according to the scores computed by Gavin and by Krogan. We also compared with a method that combines all evidences on protein-protein interactions in a naive Bayes model where motifs are not used.

Our results (Fig. 2.8(a)) show that InSite is better able to identify direct physical interactions within the top pairs. The area under the ROC curve are 0.855 and 0.916 for Prosite and Pfam respectively, while it is 0.806 for the naive Bayes model, which integrates different evidences on protein-protein interactions without using any motifs. This shows the motif based formulation is better able to provide higher rankings to the reliable direct interactions (Fig. 2.8(a)). When comparing with Gavin’s and Krogan’s scores, our model covers more positive interactions because it integrates multiple assays. However, even if we restrict only to pairs appearing in a single assay, such as Gavin’s or Krogan’s, InSite (Fig. 2.8(b,c)) is able to achieve better accuracy with either Prosite or Pfam. These results illustrate the power of using both an integrated data set and the information present in the sequence motifs in reliably predicting protein-protein interactions. A list of all protein pairs ranked by their interaction probabilities estimated by training on the full data set is available from our website [1].

2.5.3 Predicting binding sites

The key feature of InSite is its ability to predict not only that two proteins interact directly, but also the specific region at which they interact. As an example, we considered the RNA polymerase II (Pol II) complex, which is responsible for all mRNA synthesis in eukaryotes. Its 3-D structure is solved at 2.8A resolution [27], so that its internal structure is well-characterized (Fig. 2.9(a,b)), allowing for a comparison of our predictions to the actual binding sites. When using Pfam-A domains, the
Figure 2.8: **Verification of protein-protein interaction predictions.** Verification of protein-protein interaction predictions relative to reliable interactions. Protein pairs in the hidden set in a ten-fold cross validation are ranked based on their predicted interaction probabilities (green, red, and black curves for Prosite, Pfam, and naive Bayes respectively). Each point corresponds to a different threshold, giving rise to a different number of predicted interactions. The value on the X-axis is the number of pairs not in the reliable interactions but predicted to interact. The value on the Y-axis is the number of reliable interactions that are predicted to interact. The blue and brown curves (as relevant) are for pairs ranked by Gavin’s and Krogan’s scores respectively.

(a) Predictions for all protein pairs in our data set. As we can see, InSite with Pfam is better than InSite with Prosite, which is in turn better than the naive Bayes model. All those three models integrate multiple data sets and thus have higher coverage than other methods using a single assay alone. The cross and circle are the accuracies for interacting pairs based on Ito’s and Uetz’s Y2H assays respectively.

(b) Predictions only for pairs in Gavin’s assay, providing a direct comparison of our predicted probability with Gavin’s confidence score on the same set of protein pairs. The InSite model outperforms the Gavin’s score.

(c) Predictions only for pairs in Krogan’s assay, providing a direct comparison of our predicted probability with Krogan’s confidence score on the same set of protein pairs. Our InSite model outperforms the Krogan’s score.
complex gives rise to 123 potential binding site predictions: one for each direct protein interaction in the complex and each motif on each of the two proteins. Among the 123 potential predictions, 68 (55.3%) are actually binding according to the solved 3-D structure. We rank these 123 potential predictions based on our computed binding confidences. All of the top 26 predictions are actually binding (Fig. 2.9(d)). As one detailed example (Fig. 2.9(c)), Rpb10 interacts with Rpb2 and Rpb3 through its motif PF01194. We correctly predicted this motif as the binding site for the two proteins (ranked 3rd and 4th). On the other hand, there are 9 motifs on the two partner proteins that could be the possible binding sites to Rpb10. Among them, 4 are actually binding, and were all ranked among the top half of the total 123 predictions, while the other 5 non-binding motifs were ranked below 100th with low confidence score. Overall, the 6 binding sites in this example all have higher confidence scores than the 5 non-binding sites.

We performed this type of binding site evaluation for all of the co-crystallized protein pairs in PDB that also appeared in our set of gold interactions. We extracted structures from PDB that have at least two co-crystallized chains, and whose chains are nearly identical to S. cerevisiae proteins. We define two residues to be in contact if the closest distance between their two respective heavy atoms is less than 5 Å. This definition is similar to that of Koike and Takagi [73]. A motif is said to bind to a protein if they contain a residue pair that is in contact.

While the PDB data is scarce, it provides the ultimate evaluation of our predictions. We applied our method separately in two regimes. In the first, we train on Prosite motifs and evaluate on those motifs that cover less than half of the protein length (Fig. 2.10(a)); we pruned the motif set in this way because short motifs provide us with more information about the binding site location. In the second regime, we followed the protocol of Riley et al., and trained on Pfam domains and evaluated PDB binding sites on the more reliable Pfam-A domains; we also tried to both train and evaluate on Pfam-A domains but the result is worse in comparison to training on all Pfam domains (data not shown).

Overall, the PDB co-crystallized structures contain 96 potential binding sites covered by Prosite motifs, of which 50 (52.1%) are verified as actually binding, and the
Figure 2.9: Binding site predictions within the RNA Polymerase II complex.
(a) A schematic illustration of interactions within the RNA Polymerase II complex revealed by its 3-D structure. Each circle with number k corresponds to the protein ‘Rpbk’ (e.g., Rpb1).
(b) One of our top predictions is ‘Pfam-A domain PF01096 on Rpb9 binds to Rpb1’. Both Rpb9 and Rpb1 are part of the co-crystallized RNA Polymerase II complex in PDB (1I50). Rpb9 is shown as the light green chain with the surface accessible area of the domain rendered in white; Rpb1 is shown as the light orange chain with its residues that are in contact with the domain shown in orange, which verifies our prediction.
(c) Binding site predictions for interactions involving Rpb10. A red arrow connects a motif to a protein it binds to as revealed by its 3-D structure. A dashed black arrow represents a non-binding site. The numbers on the arrow are the ranks based on our predicted binding confidences. We assigned confidence values to a total of 123 motif-protein pairs in this complex. In this case, all six PDB verified binding sites (red arrows) are ranked among the top half, while all five non-binding sites have low confidence values with ranks below 100.
(d) ROC curve for our motif-protein binding sites predictions within the RNA Polymerase II complex. There are 123 possible binding sites within the complex that involves the Pfam-A domains in our data set, out of which 68 (55.3%) are actually binding according to its 3-D structure. The possible binding sites are ranked by our predicted binding confidences. The X-axis is the number of non-binding sites within the complex that are predicted to be binding. The Y-axis is the number of PDB verified binding sites that are also predicted to be binding. The purple line is what we expect by chance.
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Figure 2.10: Distribution of motif coverage.
(a) For each motif occurrence, we compute its coverage as the ratio of its length to the length of the protein it occurs on. The x-axis is the bin for the coverage. The y-axis is the number of motif occurrences that fall into this coverage bin. As we can see, most of Prosite motifs cover a small fraction of the protein while Pfam-A domains are usually longer.
(b) For each protein, we compute the fraction of its length that is covered by motif in our data set. The x-axis is the bin for the fraction. The y-axis is the number of proteins that fall into this bin. We exclude those proteins that are not covered by any motif. As we can see, if we use Prosite motifs, most proteins will have majority of their residues not covered by any motif.
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Figure 2.11: Global verification of binding site predictions. Verification of motif-protein binding site predictions relative to solved PDB structures. Possible binding sites are ranked based on our predicted binding confidences. The X-axis is the number of sites that are non-binding in PDB that are predicted to be binding. The Y-axis is the number of PDB verified binding sites that are also predicted to be binding. The green and red curve are for our InSite with Prosite and Pfam respectively, which is tailored to binding site prediction and explicitly models the noise in the different experimental assays. The brown curve is for the DPEA score as in Riley et al. The gray curve is for the score derived from the parsimony approach of Guimaraes et al.. The black curve is for the integrative approach by Lee et al.. The purple curve is what we expect from random predictions.

(a) Result using Prosite motifs. The area under the curve if we normalize both axes to interval [0,1] are 0.680, 0.601, 0.5 for InSite, DPEA by Riley et al., and random prediction respectively.

(b) Result when we train on Pfam domains and evaluate the PDB binding sites only on Pfam-A domains, as in the protocol of Riley et al. The area under the curve if we normalize both axes to interval [0,1] are 0.786, 0.745, 0.619, 0.620 for InSite, integrative approach by Lee et al., DPEA by Riley et al., and parsimony approach by Guimaraes et al. respectively.

remaining 46 are verified to be non-binding. Similarly, PDB contained 317 possible bindings between a Pfam-A domain and a protein, of which 167 (52.7%) are verified in PDB. We ranked all possible bindings according to their predicted binding confidences. With Prosite motifs (Fig. 2.11(a)), the area under ROC curve (AUC) is 0.68; note that random predictions are expected to have AUC of 0.5. For Pfam-A, when trained on all Pfam domains, we achieved an AUC of 0.786 (Fig. 2.11(b)).

We compared our results to those obtained by the DPEA method of Riley et al. [112], the parsimony approach of Guimaraes et al. [53], and an integrated approach of Lee et al. [82]. DPEA computes confidence scores between two motif types.
by forcing them to be nonbinding, and computing the change of likelihood after re-converging the model with this change. InSite differs from DPEA in two main characteristics: its confidence evaluation method, which is designed to evaluate the likelihood of binding between two particular proteins at a particular site; and the integration of multiple sources of noisy data. Guimaraes et al. use linear programming to find the confidence scores to a most parsimonious set of motif pairs that explains the protein-protein interactions. Lee et al. use the expected of number of motif-motif interactions for a pair of Pfam-A domain types across four species, and integrate them with GO annotation and domain fusion to generate a final ranking on pairs of motif types.

All of these methods generate confidence scores on pairs of motif types, regardless of what protein pairs they occur on. To use these predictions for the task of estimating specific binding regions, we define the confidence that motif M on Protein A binds to Protein B as the maximum confidence score between motif type M and all the motif types that appear on protein B. For Guimaraes et al. and Lee et al., only the confidence scores between Pfam-A domains are available so we only compared their results with our Pfam-A predictions. We re-implemented DPEA and compared with both our Prosite and Pfam-A predictions. As we can see, in both Prosite and Pfam evaluations (Fig. 2.11), the AUC obtained by InSite are the highest (0.786 and 0.680 for Pfam and Prosite respectively) while Lee et al. (0.745 for Pfam only) comes second (Kolmogorov-Smirnov p-value < 0.0002). InSite is able to reduce the error rate (1 - AUC) by 16.2% compared with Lee et al.. For Pfam, the AUC values are 0.619 and 0.620 for Riley et al. and Guimaraes et al. respectively. For Prosite, the AUC value for Riley et al. is 0.601. Compared to these two methods, InSite achieves a significant error reduction of 43.7% and 19.8% for Pfam and Prosite respectively.

If we consider the top 50 predictions made by InSite, 33 (66.0%) are correct for Prosite and 45 (90.0%) are correct for Pfam-A. In comparison, only 52.1% and 52.7% are expected to be correct using random predictions for Prosite and Pfam-A, respectively. The enrichment of known binding sites in our top predictions indicates that InSite is able to distinguish actual binding sites from non-binding sites. In comparison, the proportion of top 50 predictions verified are 82.0% (Pfam-A) for Lee
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Figure 2.12: Number of motif pair occurrences. (a) Same pair of motif types may occur between multiple pairs of proteins. Here the x-axis is the number of protein pairs, \( n \). The y-axis is the number of Prosite motif pairs that occur between exactly \( n \) pairs of proteins in our data set. (b) Same as (a), except this is for Pfam-A domains.

et al., 80.0% (Pfam-A) for Guimaraes et al., and 80.0% (Pfam-A) and 58.9% (Prosite) for Riley et al.. Note that, in the case of Pfam-A, Riley et al. predicted all top 24 pairs correctly because they are derived from the binding of PF00227 (Proteasome) with itself. This motif pair has the highest score and it appears in 24 binding events, all of which are correctly verified by PDB. The lack of granularity (i.e. pairs mediated by the same motif types have the same score) in Riley et al. helped in those top predictions, but hurt it in the remaining predictions, thus resulting in overall lower performance.
More generally, a pair of motif types may have multiple occurrences over different protein pairs (Fig. 2.12). The previous methods [53, 82, 112] assign the same confidence score to all of them. In order to demonstrate that InSite is able to make different predictions even when both motifs involved are the same, we ran InSite by forcing a pair of motif occurrences between two proteins to be non-binding and use its change of likelihood as how confident we are about whether these two motifs bind to each other. As an example, transcription factor S-II (PF01096) and RNA polymerase Rpb1 domain 4 (PF05000) are predicted to be more likely to bind when occurring between Rpb9 and Rpo31 than when occurring between Dst1 and Rpo21. This happens because there are fewer motifs on Rpb9 than on Dst1 and the motifs on Rpo31 is a subset of motifs on Rpo21. Although some alternative motif pairs between Rpb9 and Rpo31 have high affinity, overall they provide fewer alternative binding sites than those between Dst1 and Rpo21. Furthermore, Rpb9 and Rpo31 are more likely to interact than Dst1 and Rpo21. Therefore our final confidence score combines the affinity between the two motifs, the presence of other motifs on the proteins, and the interaction probability between the two proteins. Indeed, PDB verifies PF01096 and PF05000 to bind between Rpb9 and Rpo31, but not between Dst1 and Rpo21. The same reasoning applies to binding site predictions between a motif and a protein.

2.5.4 Understanding disease-causing mutations in human

While a systematic validation is not possible in human, due to the very low coverage of known protein-protein interactions or binding sites, we performed an anecdotal evaluation that focuses on interactions of particular interest for human disease. Many genetic diseases in human have been mapped to a single amino-acid mutation and cataloged in the Online Mendelian Inheritance in Man (OMIM) database [54]. The exact pathway that leads to the disease is unknown for many of the mutations. As disrupting protein-protein interaction is one way by which a mutation causes disease [119], our binding site predictions can suggest one possible mechanism for such diseases: If a mutation in protein \(A\) occurs on a motif \(M\) that is predicted to be the binding site to a protein \(B\), and \(B\) is involved in pathways related to the disease, it is
likely that the mutation disrupts the binding and thus leads to the disease. We ran InSite with two different experimental setups: one using only reliable protein-protein interactions, and the other using both reliable and high-throughput protein-protein interactions. To relate our predictions to mutations that cause human genetic diseases, we extracted the allelic variants from OMIM [54], which describes where the mutations occur and their related diseases. We get a total of 737 mutations covering 131 motifs in 97 proteins of our training data.

Table 2.1 lists our top ten predictions from each experiment with relevant literature references. As in yeast, we exclude those motifs that cover more than half the length of the protein, so we focus on short motifs that provide us with more information about the binding site. Note that eight predictions are among the top ten in both experiments, showing the robustness of our method to different protein-protein interaction data. A full list of our predictions is available from our website [1].

Some of our predictions are directly validated in the literature. One of the top ten predictions involves Vitamin K-dependent protein C precursor PROC, which is predicted to bind to Vitamin K-dependent protein S precursor PROS1. There are four regions on PROC — Gla domain, EGF-like domain 1, EGF-like domain 2, and Serine proteases domain. Prosite has ten motifs on the protein, covering those four regions. InSite predicted two of the motifs (PS01187 and PS50026), which correspond to EGF-like domain 1, to be the binding site to PROS. Ohlin et al. [105] showed that antibody binding to the region of the EGF-like domain 1 reduces the anticoagulant activity of PROC, apparently by interfering with the interaction between activated protein C and its cofactor PROS1. Therefore, they propose the domain to be the binding site on PROC with PROS, thus validating our prediction. A mutation in the domain causes thromboembolic disease due to protein C deficiency [2], matching the fact that defects in PROS1 are also associated with an increased risk of thrombotic disease (Uniprot:P07225). These facts support a hypothesis in which the mutation on PROC leads to the disease by disrupting the interaction with PROS1.

Another of our highest-confidence binding site predictions is: ‘the BH3 motif on BAX binds to BCL2L1’ (Fig. 2.13). BCL2 has inhibitory effect on programmed cell
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<table>
<thead>
<tr>
<th>Protein</th>
<th>Partner</th>
<th>Binding site</th>
<th>OMIM disease</th>
<th>Pubmed</th>
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Table 2.1: **Top binding site predictions in human.** We list the top 10 binding site predictions in human that contain disease causing mutation. The top panel is the predictions when using only reliable protein-protein interactions. The bottom panel is the predictions when integrating high-throughput interactions. Eight predictions appear in both panels, showing our method is robust to the change in the input data. Shown are the protein, its interacting partner, the motif that is predicted to be the binding sites to its partner, the disease caused by the mutations inside the motif, and the Pubmed reference to the interaction. Three of top predictions are verified by literature (bold and italic), four in the top panel and three in the bottom panel are supported by existing evidence (bold), one in the top panel and two in the bottom panel are confirmed to be wrong (italic), and the remaining two predictions do not have literature information. In some cases, it is possible that the mutations at the binding site disrupt the interaction, and thus lead to the disease.
Figure 2.13: Illustrations of human binding site predictions. Schematic representation of our top prediction and its validation by literature. BAX has four motifs: BH3 motif (PS01259), BH1 (PS01080), BH2 (PS01258), and BCL2-like apoptosis inhibitors family profile (PS50062). BH3 (in red) has the highest change in log-likelihood among those motifs, and is among one of our top predictions (1.9%). Reed et al. (1996) confirmed that BH3 on BAX is involved in binding with BCL2. On the other hand, the binding site on BCL2 involves portions where all of BH1, BH2, and BH3 reside. Interestingly, none of these motifs on BCL2L1 have high confidence to be binding site, with the highest one also being BH3 and ranked in the top 8.7%. Mutations in BAX (in position shown by the black bar) cause leukemia.
death (anti-apoptotic) [62] while BAX is a tumor suppressor that promotes apoptosis. Approximately 21% of lines of human hematopoietic malignancies possessed mutations in BAX, perhaps most commonly in the acute lymphoblastic leukemia subset [94]. There are four motifs on BAX (Fig. 2.13) and we predict BH3 to be the binding site to BCL2 with high confidence (top 1.9%). By searching the literature, we found that Zha et al. [141] showed that the BH3 motif on BAX is involved in binding with BCL2, thus validating our binding site prediction. However, BH3 is also required for homo-oligomerization of BAX, which is necessary for the apoptotic function [46]; thus, the BH3 mutation may cause the disease by disrupting the BAX homo-oligomerization. From the BCL2 side, the associated binding site involves the portion where three motifs — BH1, BH2, and BH3 — reside [110]. If we examine the InSite binding site predictions on BCL2, none of the motifs is predicted to have high confidence, with the best one — BH3 — ranked at the 8.7th percentile. Therefore, InSite has the flexibility to predict the binding site in one direction, but not the other direction.

Some of our predictions (Table 2.1) are not directly verified but are consistent with existing literature evidence, and provide biologists with testable hypotheses for possible further investigation. As one example, a mutation at codon 404 in MMP2 causes Winchester syndrome [2]. However, it is not well understood how diminished MMP2 activity leads to the changes observed in the disease [140]. InSite predicted the zinc-binding peptidase region on MMP2, which contains codon 404, to be the binding site to BCAN. As BCAN is degraded by MMP2 [101], the peptidase region we predicted is likely to be the binding site that catalyzes the degradation of BCAN. Codon 404 is believed to be essential for the peptidase activity [2], consistent with our hypothesis that its mutation might disrupt the interaction between MMP2 to BCAN. Our binding site prediction provides one possible hypothesis that implicates BCAN in the process of pathogenesis.

We also listed all top predictions that are confirmed to be wrong (Table 2.1). In one case, the prediction involves the Ephrins signature, which is an example of a ‘signature motif’. Such motifs represent the most conserved region of a protein family or a longer domain, and are used by Prosite to conveniently identify the longer
domain. InSite cannot distinguish the behavior of the signature from the domain. Therefore, when the signature motif is predicted to be the binding site, the actual binding could take place in the longer domain. In the case of the Ephrins signature, Prosite uses the motif to identify the Ephrins protein family. Therefore, we would not generally expect a binding site to overlap the motif.

In a similar validation to our OMIM analysis, we considered a recent data set by Greenman et al. [52] produced by screening protein kinases for mutations associated with cancer. However, in many cases, it is unknown whether a mutation is a driver mutation that causes the cancer, or whether it is a passenger mutation that occurs by chance in the cancer cell. Even for driver mutations, the mechanism by which it leads to cancer is often unknown.

To relate our predictions to mutations in cancer, we extracted more than 1,000 somatic mutations found in 274 megabases (Mb) of DNA corresponding to the coding exons of 518 protein kinase genes in 210 diverse human cancers [52]. We focused only on those proteins that are predicted to contain a driver mutation. This results in a total of 652 mutations covering 489 motifs in 249 proteins of our training set.

We considered those mutations that fall in InSite predicted binding sites. Among all the potential driver mutations identified by Greenman et al., the one most likely to be a binding site according to the InSite predictions is the SH2 domain of FYN in SRC family (Fig. 2.14), which is predicted to bind to proto-oncogene vav (VAV1). Greenman et al. found three mutations on FYN and predicted with 0.985 probability that at least one of them is a driver mutation [52]. This finding suggests the hypothesis that the mutation disrupts the binding of SH2 domain to VAV1, and thus causes cancer. Indeed, a literature search shows that the SH2 domain on FYN is known to bind to VAV1 [99], thereby validating our binding site prediction. Moreover, VAV1 was discovered when DNA from five esophageal carcinomas were tested for their transforming activity [5], which is compatible with the fact that FYN is implicated in squamous cell carcinoma [52]. These observations support the disruption of the FYN-VAV1 binding as the cause for the disease in this case.
Figure 2.14: **3-D structure of one of our top predictions.** A fragment of FYN with SH2 and SH3 domain is crystallized in PDB (ID: 1G83) and is visualized here. The fragment accounts for about 30% of the total protein length and is rendered in a ribbon representation. The SH2 domain, which is colored in green, is predicted to be the binding site to VAV1. The position of the potential driver mutation found in somatic cancer cell is highlighted by the white balls.
2.6 Discussion

Obtaining computational models for the mechanism of protein-protein interactions is an important but challenging task. Other computational methods for discovering protein-protein interaction sites fall into two broad categories. The first are docking methods that try to match two protein structures to find the best sites on both structures [51]. These methods only apply to solved protein structures, which are currently available only for a small number of proteins. To enlarge the set of applicable proteins, some methods [69, 90, 8, 93] use homology to proteins with known structures, but many proteins do not, as yet, have any homologous with solved structure, necessitating the use of other techniques. The second class of methods use local sequence information to predict interaction sites [104, 73]. These methods typically train a machine learning algorithm (such as a neural network) to identify interaction sites, and therefore require solved complexes to provide examples of interaction sites as training data. As such examples are relatively scarce, the available data might not sufficiently capture the sequence variability found in interaction sites, which can lead these methods to have low sensitivity. Our approach uses only the widely-available sequence information and raw protein-protein interaction data, and therefore offers the promise of identifying binding sites on a genome-wide scale.

InSite is able to integrate different sources of assays in a principled way and learn a different observation model for each assay. InSite explicitly models the noise from high-throughput assays and the possibility that two proteins in the same complex do not physically interact. This allows us to use the noisy data as well as assays aimed at identifying complexes, so our interaction data set is much bigger than any that have been used before, providing both higher coverage and increased robustness. Our data integration method is unique in not utilizing a ‘gold standard’ set of interactions (such as ones obtained from low-throughput experiments) for training, thereby greatly increasing the size of the training set and avoiding possible biases in the training set. InSite also easily accommodates other types of indirect evidence, such as co-expression, GO annotation, and domain fusion, on both protein-protein interactions and motif-motif interactions. This type of integration may be useful in
other settings as well (see Section 5). We note that the evidence model, although an important component in our approach, is not the main factor in its performance. Indeed, if we remove the indirect evidence like co-expression, GO annotation, and domain fusion from our model, the AUC values decreases by only 0.033 and 0.019 for Pfam and Prosite respectively (Fig. 2.15). Therefore our result using protein-protein interactions alone is still significantly better than the methods of Guimaraes et al. and Riley et al., which also only rely on protein-protein interaction, and it beats Lee et al., which uses multiple types of data including indirect evidences. On the other hand, if we add our evidence model onto the model of Riley et al., the AUC values increase by only 0.017 and 0.009 for Pfam and Prosite respectively. Therefore, the main component in the performance of our model is the construction of predictions that are targeted at specific protein pairs and take their particular context into account.

There are several limitations to the ability of our approach to identify correct binding sites. Not all motifs mediate protein interactions through direct binding. Some motifs help shape the structure of the proteins. Mutations in the motifs would alter the structure of the protein and disrupt the bindings at some other places. Other motifs are signatures that are markers for longer domains. It is the longer domain, and not the signature motif, that serves as the actual binding site. InSite will not be able to distinguish these cases. One approach would be to classify motifs into either structural or binding motifs by using partially supervised learning with labeled binding sites from PDB or prior biological knowledge. A motif may appear multiple times in a protein, but InSite is unable to distinguish between them, and therefore cannot predict which copy is the actual binding site. Most importantly, some binding sites may not be covered by any motif in our set of conserved motifs (Fig. 2.7 and Fig. 2.10(b)), and thus our current model has no way to predict interactions involving them. Clearly, we can apply InSite to a larger set of motifs, e.g., eMotifs [125, 60], but there may still be motifs that cannot be identified by conservation. Thus, the most significant extension of our method would be to allow it to search for a motif in cases where there is no pre-existing motif that provides a good explanation for the observed interactions. One possible approach may be an integration of InSite with approaches that use sequence to predict binding sites directly [104, 73].
Figure 2.15: **Contribution of indirect evidence.** Verification of motif-protein binding site predictions relative to solved PDB structures. Possible binding sites are ranked based on our predicted binding confidences. The X-axis is the number of sites that are non-binding in PDB that are predicted to be binding. The Y-axis is the number of PDB verified binding sites that are also predicted to be binding. The green (Prosite) and red (Pfam) curve are for our InSite applied to both protein-protein interaction assays and indirect evidences on protein-protein interactions and motif-motif interactions, such as co-expression, GO distance, and domain fusion. The blue curve is for InSite applied to only protein-protein interaction assays. The purple curve is what we expect from random predictions.

(a) Result using Prosite motifs. The area under the curve if we normalize both axes to interval [0,1] are 0.680, 0.661, 0.500 for InSite applied to all data, InSite applied only to protein-protein interactions, and random predictions respectively.

(b) Result when we train on Pfam domains and evaluate the PDB binding sites only on Pfam-A domains, as in the protocol of Riley *et al.* The area under the curve if we normalize both axes to interval [0,1] are 0.786, 0.754, and 0.500 for InSite applied to all data, InSite applied only to protein-protein interactions, and random predictions respectively.
2.7 Conclusions

In the past few years, there is a growing suite of methods that successfully utilize large amounts of available data and sophisticated machine learning methods to solve problems in structural biology for which experimental methods are difficult and time-consuming. These tasks include protein structure prediction [15], RNA structure prediction [33, 113], side-chain prediction [136], protein surface prediction, and more. Following in this tradition, we have developed InSite, a novel probabilistic method for predicting regions at which two interacting proteins bind to each other. InSite makes use of three types of data sets: direct protein-protein interaction assays; indirect evidence on protein-protein interactions such as co-expression; and indirect evidence on motif-motif interactions such as domain fusion. It provides a principled integration of these data sets, which may be noisy, and may not correspond to direct physical interaction. In future work, the flexibility of the framework would allow us to easily extend it to include more types of information, including structural information. For example, we can use motif-motif bindings in PDB to construct a more informed prior for the motif-motif affinity.

InSite makes targeted, testable predictions for specific binding regions in an interacting protein pair. As we have shown, these predictions can be used to generate hypotheses regarding the mechanism by which certain mutations in a protein can disrupt interactions, and give rise to phenotypic changes, including human disease such as cancer. We put all predictions with cancer annotation or OMIM mutation online, allowing a more comprehensive analysis by experts and follow-on wet-lab experiments. We have also made the InSite software publicly available via the web to allow this tool to be used by researchers. Due to the universal mechanisms underlying biochemical interactions, the tool can be applied to any organisms, and even to protein-protein interaction data generated from multiple organisms.
Chapter 3

MRFs: modeling interaction and complex

Many protein-protein interactions occur between proteins in the same complex. In this chapter, we exploit that relationship to improve the prediction accuracy of protein-protein interactions and to predict complexes directly. We use the framework of Markov Random Fields (MRFs), which enables us to encode the relationship between different entities and make predictions on all of them at the same time. We develop an algorithm for fast inference in MRF so we can apply our model to the entire proteome.

3.1 Introduction

Many of the protein-protein interactions we observe in the previous chapter are derived from proteins in the same complex: if protein $A$ interacts with $B$ and $B$ interacts with $C$, it is likely $A$, $B$, and $C$ are in the same complex and thus $A$ also interacts with $C$. This transitivity relationship suggests that instead of predicting the interaction between each pair of proteins independently, we can try to predict all of them ‘collectively’ at the same time by exploiting the correlation among them, as Jaimovich et al. [64] did using an MRF model. We built on the work of Jaimovich et al. to also
take into account relationships that involve other type of data; for example, if \( A \) transcriptionally regulates both \( B \) and \( C \), then \( B \) and \( C \) are more likely to interact. We demonstrate how to do this and improve the prediction accuracy on protein-protein interactions in the first part of this chapter.

The task of ‘collective classification’, where a set of labels are predicted together while considering their dependencies, fits well into the framework of MRFs. An MRF, like a Bayesian Network, is a kind of probabilistic graphical model. It is a powerful framework and a principle way to encode prior domain knowledge about the relationship between different entities. It allows us to collectively predict all the unknown entities while taking into consideration the correlation between the predictions, such as the transitivity relationship.

In particular, we applied the MRF model to the problems of predicting all interactions between proteins. We construct an MRF that consists of nodes representing transcriptional regulation, protein-protein interactions, localization, and observed values for these underlying biological entities based on high-throughput experimental assays. Edges and cliques in the network encodes the transitivity relationship among protein-protein interactions and between protein-protein interactions and transcriptional regulation. The MRF model is also able to effectively deal with the noise in the experimental assays, and correlate the protein-protein interaction with co-localization.

The transitivity relationship we use is largely a result of multiple proteins associating with each other to form a complex. So instead of focusing on protein-protein interactions, why not directly predicting the underlying biological entity — protein complexes? This would also avoid an intrinsic bias with the triplet model described in Section 3.2. A complex, which is stoichiometrically stable, is a basic biological unit that has its own properties and serves as the building block of high level structures. Therefore, identifying a list of complexes is a key middle step for our understanding of the mechanism from proteins to functions. The recent technology of tandem affinity purification followed by mass spectrometry (TAP-MS) provided us with large amounts and high quality measurements of co-complexed proteins. With the TAP-MS scores, it becomes possible for a genome-wide reconstruction of complexes. The MRF, which is a flexible framework, can be readily applied to construct a model for
protein complexes, as we demonstrate in the second part of this chapter. We will later design better methods for the same task in the next chapter.

In this case, the node in the MRF is a protein-complex pair, \((P, C)\), where the node has value 1 if protein \(P\) is in complex \(C\) and has value 0 if \(P\) is not in \(C\). We then connect nodes for the same complex and use the potential function to encode whether we prefer those two proteins to be both in that particular complex. The potential function depends on the pairwise features between those two proteins and the weights of those features. The pairwise features come from different types of evidence, such as the TAP-MS score and co-localization, between the two proteins. They provide signals to whether these two proteins are likely to be in the same complex or not. The potential function is a linear combination of the features, weighted to account for their relative contribution appropriately. The larger the potential, the more likely the two proteins are both in the same complex. We learn those weights by maximizing the joint likelihood of the MRF, which is instantiated on the reference complexes. We use maximum a posteriori (MAP) inference to identify new complexes.

There is a vast amount of research devoted to efficient learning and inference in probabilistic graphical models in general, and MRFs in particular. However, most approaches are slow and approximate, which severely limits the applications of MRF. In recent years, a new algorithm is developed that converts the inference problem in an MRF into a minimum cut (mincut) problem in a directed graph, which can be solved efficiently using the maxflow algorithm \([75]\). The inference algorithm based on mincut is fast and exact but is only limited to a special class of MRF that has all regular (submodular) potentials, which require neighboring nodes to be more likely to have the same value. For the triplet model, all the potentials are close to being regular. Therefore, we constrain them to be regular during the learning so that we can apply the fast and exact mincut inference. The results show that we improve prediction accuracy of protein-protein interactions by incorporating more types of evidence and doing collective classification so the correlations between all entities are taken into consideration. For the complex model, however, the edge potential between two proteins would not be regular if the proteins are less likely to be in the same complex based on their features. Therefore, we extended the mincut inference
to accommodate non-regular potentials. The new algorithm, while still being fast, can be applied to a wide range of MRFs, including ones that represent interesting problems in biology. We applied it to our complex models and its predictions of complexes achieved high sensitivity. For both the triplet model and complex model, the speedup of the inference enables us to work on the entire genome.

3.2 Related work

Kolmogorov and Zabih [75] applied mincut/maxflow algorithms to solve the problem of MAP inference. It is fast and exact. However, it only applies to a special class of MRFs that have all binary variables with regular potentials. To deal with variables of more than two values, Boykov et al. [17] developed an $\alpha$-expansion algorithm that converts a non-binary MRF into solving a series of binary MRFs, where the energy of the original MRF is guaranteed to increase at each iteration. To deal with MRFs with non-regular potentials, Kolmogorov and Rother [74] described a method called QPBO that extends the mincut inference. However, its output is only a partial assignment and there may be variables that remain unresolved. In order to get a full assignment to all the variables, Cremers and Grady [28] dealt with non-regular terms by throwing away potential functions that do not satisfy the regularity constraint Eq. (3.2). The resulting MRF would be regular, and efficient exact inference can be performed. However as shown in [74], the performance degrades when many non-regular potentials are needed for the problem. Rother et al. [114] proposed to ‘truncate’ the non-regular terms, i.e. replacing them with regular approximations, so that at each iteration of the $\alpha$-expansion algorithm, which solves the multi-class labeling problem, the energy is guaranteed to decrease. We propose an algorithm that combines limited ‘truncation’, i.e. replacing a subset of non-regular potentials with regular approximations, with the QPBO method that does partial inference. This procedure resolves more variables than QPBO, and in certain cases produces a provably correct assignment. We also propose a procedure that gives an approximate assignment to unresolved variables if there are any left.

The MAP inference can also be formulated and solved as an integer programming
problem, whose LP relaxation was first proposed by [116], and then subsequently
rediscovered independently by several researchers. Komodakis and colleagues [76, 77]
make use of the dual of the LP relaxation in the context of graph cut methods, where
it corresponds to the well-known duality between min-cut and max-flow. They use
this approach to derive primal-dual methods that speed up and extend the alpha-
expansion method in several ways.

Jaimovich et al. [64] proposed several MRF models for protein-protein interactions,
which use observed and hidden localization and interaction variables to capture the
transitivity properties described above. There is, however, an intrinsic bias with his
models. The protein-protein interaction nodes belong to two groups: those with value
1 are pairs of proteins that are known to interact; those with value 0 are randomly
picked protein pairs that are assumed to be non-interacting. If we only randomly pick
the same number of non-interacting nodes as known interactions, like what Jaimovich
et al. did, we end up with a sparse set of protein pairs. It is unlikely that any three
nodes in the set would form a triplet. On the other hand, a known interaction, such
as $A - B$, is often part of a larger complex, say $(A, B, C)$. It is likely to form triplets
with other known interactions in the complex, such as $A - C$ and $B - C$. In general,
the nodes with value 1, representing known interactions, are likely to form triplets
with each other. Therefore, our model only needs to learn a simple classifier that
predicts a node to be 1 (interacting) if it is part of a triplet and 0 (non-interacting)
if it is not. We address this problem by picking many more non-interacting nodes
than known interactions so there are enough triplets involving both types of nodes:
non-interacting pairs and known interactions.

In order to perform ‘collective classification’ over a set of protein pairs, Jaimovich
et al. [64] used Loopy Belief Propagation (LBP) [107, 137] to do the inference, which
is approximate and slow. Due to the long running time of LBP, they were limited
to predicting interactions between a small subset of all proteins: 543 of the approxi-
mately 6000 proteins in yeast. We extend their model to include regulation variables
and use fast inference to reduce the running time.

There are other works that apply MRFs to protein-protein interaction network.
For example, works by Letovsky and Kasif [87] and Segal et al. [118] used MRFs to
encode the fact that interacting proteins are more likely to share the same function or be in the same functional module, i.e., neighbors in a protein-protein interaction network should be more likely to have the same functional assignment. Therefore, they made accurate predictions of protein functions or functional module assignments by doing inference in the network.

There are many works on reconstructing complexes. For a detailed review, see the Related Work section of the next chapter (Section 4.2).

3.3 Background

3.3.1 Markov Random Field (MRF)

In many biological problems, the correlation between the labels is important. In those cases, it is helpful to make predictions on all the instances at the same time while taking into considerations the correlations. For example, when predicting protein-protein interactions, if our evidence strongly indicates that proteins $A$ and $B$ interact and proteins $B$ and $C$ interact, we are more likely to believe that proteins $A$ and $C$ also interact based on transitivity. This kind of structure or other domain knowledge can be encoded in a principled way through an MRF.

An MRF, also called Markov network, is an undirected probabilistic graphical model, as opposed to a Bayesian network, which is a directed model. It is a way to compactly define a joint distribution over a set of random variables. It encodes the conditional independence relationships by the structure of the graph and its parameters are associated with the local neighborhoods of the graph. Algorithms have been developed to learn both the structure of the graph, which defines the conditional independences, and the parameters, which quantitative the joint distribution. Given a learned model and some observed variables, we can obtain the marginal distribution or the most likely assignment to the unobserved variables.

Representation of MRF

An MRF is defined as:
1. an undirected graph $G = (V, E)$, where each node $X \in V$ represents a random variable and edges between the nodes represent probabilistic dependencies.

2. a set of potential functions defined over the cliques (complete subgraphs) in $G$.
   A potential function $\phi_c(X_c)$, where $X_c$ are the variables in clique $c$, maps every assignment to $X_c$ to a non-negative real number.

The joint distribution defined by the above Markov network is:

\[
P(x) = \frac{1}{Z} \prod_{c \in C} \phi_c(x_c)
\]

where $x$ is a particular assignment to all variables $X$, $C$ is the set of all the cliques, $x_c$ is the assignment to the variables in clique $c$ induced by the joint assignment $x$, and $Z$ is the normalizing constant to make sure $P(X)$ is a probability distribution:

\[
Z = \sum_x \prod_{c \in C} \phi_c(x_c)
\]

If two variables are connected in the graph, they are dependent on each other, and their dependency is specified in the potential function. On the other hand, if they are not connected, they are conditionally independent given all the other variables.

**Inference in MRF**

There are two main types of inference tasks. Marginal inference computes the marginal distribution for variables. Maximum a posteriori (MAP) inference computes the most likely assignment of a set of variables. For a general network, these inference tasks are NP-hard. Algorithms based on belief propagation (BP) [107, 137] have shown good promise in doing marginal inference. However, the BP algorithm can still be computationally intensive and is not guaranteed to converge. Recently, there has been great progress in MAP inference by using mincut and maxflow algorithms. However until recently, they can only be applied to MRFs with special type of potential functions. In this thesis, we extend the algorithm to do approximate MAP inference over any MRF. It is shown to be fast and achieved good accuracy.
The basic concept of MAP inference using graph cut is to convert a MRF into an specialized graph such that the most likely assignment in the MRF corresponds to the minimum cut (mincut) in the graph. We then solve the mincut problem using maxflow algorithm, which is very efficient. The resulting solution can be easily mapped back to the MAP assignment in the MRF. However, the conversion is only possible for a special class of potential function.

In particular, our MAP inference objective is:

\[
\begin{align*}
\arg\max_x P(x) &= \arg\max_x \prod_{c \in C} \phi_c(x_c) \\
&= \arg\min_x \sum_{c \in C} \psi_c(x_c) \\
&= \arg\min_x Energy(x)
\end{align*}
\]

(3.1)

where \(\psi_c = -\log \phi_c\). In order to maximize the likelihood, we just need to minimize the energy.

We only consider MRFs where every clique \(c\) involves no more than three variables, which covers most MRFs we encounter in biological domains. The corresponding mincut graph is a directed graph with positive edge weights. It has a vertex \(v_i\) for each node \(x_i\) in the MRF, in addition to two terminal vertices: source \(s\) and sink \(t\). A cut of the graph is a partition of the vertices into \(S\) and \(T\), where \(s \in S\) and \(t \in T\). The cost of the cut is the sum of all edges going from \(S\) to \(T\). Next we describe how we convert the potential functions of the MRF to the edges in the mincut graph based on Kolmogorov and Zabih [75].

**Node clique: one variable** For each node term in Eq. (3.1), \(\psi_i(x_i)\), we compute \(\delta_i = \psi_i(x_i = 0) - \psi_i(x_i = 1)\). We add an edge from \(s\) to \(v_i\) of weight \(-\delta_i\) if \(\delta_i < 0\) and an edge from \(v_i\) to \(t\) of \(\delta_i\) if \(\delta_i > 0\).

**Pairwise clique: two variables** For each pairwise term in Eq. (3.1), \(\psi_{ij}(x_i, x_j)\), we decompose it into two node terms and a pairwise term with only one non-zero
\[
\psi_{i,j}(x_i, x_j) = \begin{bmatrix}
\psi_{i,j}(0,0) & \psi_{i,j}(0,1) \\
\psi_{i,j}(1,0) & \psi_{i,j}(1,1)
\end{bmatrix} = \begin{bmatrix}
A & B \\
C & D
\end{bmatrix}
\]

Table 3.1: Representing a pairwise term.

\[
\begin{bmatrix}
A & B \\
C & D
\end{bmatrix} \begin{bmatrix}
A & A \\
C & C
\end{bmatrix} + \begin{bmatrix}
0 & D-C \\
0 & D-C
\end{bmatrix} + \begin{bmatrix}
0 & B+C-A-D \\
0 & 0
\end{bmatrix}
\]

Table 3.2: Decomposition of a pairwise term. Decomposition of a pairwise term into the sum of two node terms and a pairwise term with only one non-zero component.

\[
\psi_{i,j}(x_i, x_j) = N_1(x_i) + N_2(x_j) + E(x_i, x_j)
\]

where:

\[
\begin{align*}
N_1(0) &= \psi_{i,j}(0,0) \\
N_1(1) &= \psi_{i,j}(1,0) \\
N_2(0) &= 0 \\
N_2(1) &= \psi_{i,j}(1,1) - \psi_{i,j}(1,0) \\
E(0,1) &= \psi_{i,j}(0,1) + \psi_{i,j}(1,0) - \psi_{i,j}(0,0) - \psi_{i,j}(1,1) \\
E(0,0) &= E(1,0) = E(1,1) = 0
\end{align*}
\]

Table 3.1 is a convenient way to represent \(\psi_{i,j}(x_i, x_j)\) and Table 3.2 illustrates the above decomposition.

We convert the two node terms into edges in the mincut graph as we describe in the previous section. We add an edge from \(v_i\) to \(v_j\) with weight \(\psi_{i,j}(0,1) + \psi_{i,j}(1,0) - \psi_{i,j}(0,0) - \psi_{i,j}(1,1)\). Since the edge weight has to be positive, this conversion is valid only if:
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$\psi_{i,j,k}(x_i, x_j, x_k) = \begin{bmatrix} \psi_{i,j,k}(0, 0, 0) & \psi_{i,j,k}(0, 0, 1) \\ \psi_{i,j,k}(0, 1, 0) & \psi_{i,j,k}(0, 1, 1) \\ \psi_{i,j,k}(1, 0, 0) & \psi_{i,j,k}(1, 0, 1) \\ \psi_{i,j,k}(1, 1, 0) & \psi_{i,j,k}(1, 1, 1) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \\ E & F \\ G & H \end{bmatrix}$

Table 3.3: Representing a triplet term.

$\begin{array}{c|c|c} A & B \\ C & D \\ E & F \\ G & H \end{array} = \begin{array}{c|c|c|c} A & A \\ C & C \\ E & E \\ G & G \end{array} + \begin{array}{c|c|c|c} 0 & B - A \\ 0 & B - A \\ 0 & F - E \\ 0 & F - E \end{array} + \begin{array}{c|c|c|c} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & P \end{array}$

Table 3.4: Decomposition of a triplet term. Decomposition of a triplet term into the sum of three pairwise terms and a triplet term with only one non-zero component.

$\psi_{i,j}(0, 1) + \psi_{i,j}(1, 0) - \psi_{i,j}(0, 0) - \psi_{i,j}(1, 1) \geq 0$

$\iff$

$\psi_{i,j}(0, 1) + \psi_{i,j}(1, 0) \geq \psi_{i,j}(0, 0) + \psi_{i,j}(1, 1)$

$\iff$

$\phi_{i,j}(0, 0)\phi_{i,j}(1, 1) \geq \phi_{i,j}(0, 1)\phi_{i,j}(1, 0) \quad (3.2)$

We call potential functions satisfying the above condition regular.

**Triplet clique: three variables** We conveniently represent the triplet term in Eq. (3.1), $\psi_{i,j,k}(x_i, x_j, x_k)$, as in Table 3.3. We define:

$$P = B + C + E + H - A - D - F - G$$

If $P \leq 0$, we decompose the triplet term as shown in Table 3.4. The first box on the right side is a pairwise term independent of $x_k$. The second and third boxes are pairwise terms independent of $x_j$ and $x_i$ respectively. They can be converted into the edges in the mincut graph as we described earlier as long as
Figure 3.1: MinCut graph for a triplet term without pairwise components. Any triplet term can be decomposed into the sum of three pairwise terms plus a residue triplet term, which can be converted to one of the above mincut graph:

(a) Edge weights are all $-P$. The cost of the minimum cut is 0 if $x_i = x_j = x_k = 1$ and $-P$ otherwise.

(b) Edge weights are all $P$. The cost of the minimum cut is 0 if $x_i = x_j = x_k = 0$ and $P$ otherwise.

they satisfy the regularity condition:

\[
\begin{align*}
C + E & \geq A + G \\
B + E & \geq A + F \\
B + C & \geq A + D
\end{align*}
\]

To represent the last box, we will add an auxiliary vertex $u_{ijk}$ and four edges of weight $-P$: $v_i \rightarrow u_{ijk}$, $v_j \rightarrow u_{ijk}$, $v_k \rightarrow u_{ijk}$, and $u_{ijk} \rightarrow t$ (Fig. 3.1(a)). It is easy to verify that the cost of the minimum cut is 0 if $x_i = x_j = x_k = 1$ and $P$ otherwise, which is equivalent to the last box by a constant.

In the case of $P > 0$, we decompose it as shown in Table 3.5.
Table 3.5: Decomposition of a triplet term. Decomposition of a triplet term into the sum of three pairwise terms and a triplet term with only one non-zero component.

The first three boxes on the right side are pairwise terms, which can be converted into the edges in the mincut graph as we described earlier as long as they satisfy the regularity condition:

\[
\begin{align*}
D + F & \geq B + H \\
D + G & \geq C + H \\
F + G & \geq E + H
\end{align*}
\]

To represent the last box, we will add an auxiliary vertex \(u_{ijk}\) and four edges of weight \(P\): \(u_{ijk} \rightarrow v_i, \ u_{ijk} \rightarrow v_j, \ u_{ijk} \rightarrow v_k, \ s \rightarrow u_{ijk}\) (Fig. 3.1(b)). It is easy to verify that the cost of the minimum cut is 0 if \(x_i = x_j = x_k = 0\) and \(P\) otherwise, which is equivalent to the last box by a constant.

It is easy to verify that the cost of any cut \(S,T\) would equal to the energy of the MRF, up to a constant, if we assign \(x_i = 0\) for \(v_i \in S\) and \(x_i = 1\) for \(v_i \in T\). Therefore, our problem becomes finding the minimum cost cut for the graph, which can be efficiently solved by using the maximum flow algorithm (maxflow) [6].

Kolmogorov and Rother [74] described a method called QPBO that extends the mincut inference and deals with MRFs with non-regular potentials. According to the QPBO method, in addition to the terminal vertices \(s\) and \(t\), there are two vertices \(v_i\) and \(w_i\) for each node \(x_i\) in MRF. The edges involving \(v_i\) would be constructed exactly the same way as described above, except all the weights are halved. The edges involving \(w_i\)’s are reversed from those of the \(v_i\)’s:

- If there is an edge from \(v_i\) to \(v_j\), we add an edge from \(w_j\) to \(w_i\).
- If there is an edge from \(v_i\) to \(t\), we add an edge from \(s\) to \(w_i\).
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\[
\begin{pmatrix}
0 & r \\
0 & 0
\end{pmatrix}
= \begin{pmatrix}
r & r \\
0 & 0
\end{pmatrix}
+ \begin{pmatrix}
-r & 0 \\
0 & 0
\end{pmatrix}
\]

Table 3.6: **Rewrite a non-regular pairwise term.** Any non-regular potential can be re-written as the sum of regular terms and pairwise non-regular terms with only the \((0,0)\) component being non-zero \((-r > 0)\).

- If there is an edge from \(s\) to \(v_i\), we add an edge from \(w_i\) to \(t\).

The idea is that if we run mincut only in the part of \(w_i\)’s, we will get the exact reverse partition, i.e. \(w_i \in T\) iff \(v_i \in S\). Given a regular MRF, since the two parts are disconnected, except at \(s\) and \(t\), we can run the mincut on the whole graph and assign node \(x_i\) to 0 iff \(v_i \in S\) and \(w_i \in T\).

For any non-regular MRF, we can still do the same decomposition as described above, except we will be left with some pairwise term \(E(x_i, x_j)\) with only one non-zero component, \(r = E(0,1)\), which cannot be converted into the mincut graph because \(r < 0\). We further decompose it as in Table 3.6.

The first box on the right side represents a node term on \(x_i\) and can be converted to edge weights in the mincut graph based on the procedure described earlier. For the second box, we add two edges from \(v_i\) to \(w_j\) and \(v_j\) to \(w_i\) with weight \(-r/2\) for each of them. It is easy to verify that for any cut satisfying \(v_i \in S\) iff \(w_i \in T\), the cost of the cut would be the same as the energy in the corresponding MRF, with node \(x_i = 0\) iff \(v_i \in S\) and \(w_i \in T\).

However, the result of the mincut algorithm may not put every pair of \((v_i, w_i)\) in a different partition, which may cause the node \(x_i\) in the MRF to be unresolved. At the end of the algorithm, \(x_i\) is assigned to be 0 if \(v_i \in S\) and \(w_i \in T\); \(x_i\) is assigned to be 1 if \(v_i \in T\) and \(w_i \in S\); \(x_i\) remains unresolved otherwise.

Kolmogorov and Rother [74] proposed a procedure that will have most of the \(X\) variables resolved among the different cuts that all achieve the minimum cost. They also showed that there exists a MAP assignment that matches the assignment to all the resolved variables in \(X\). This is true even if there are some other variables that remain unresolved. This implies that we can get provably correct partial assignment for any MRFs, including those with non-regular potentials.
Learning in MRFs

Learning algorithms have been developed that takes either the marginal inference or MAP inference as a subroutine. Therefore, it is important to have a fast and accurate inference algorithm that can be applied to any kind of MRF. Maximum likelihood learning, which optimize the likelihood by gradient descent, uses marginal inference to compute the gradient. The marginal inference is usually done with loopy belief propagation, which is slow and approximate. Max-Margin Markov Network learning [126], which tries to maximize the margin between the true labels and all the other labels, uses MAP inference to generate constraints in its QP optimization problem.

In this thesis, we first used maximum likelihood learning, but with the MAP assignment as the gradient, which approximates the marginals but is a magnitude faster than computing the marginals using LBP. Since the gradient is no longer continuous, our optimization algorithm would stop earlier before reaching the actual global maximum. We then tried the perceptron learning with voting [24], which also takes MAP inference as a subroutine. In both cases, we used our fast MAP inference algorithm, which greatly speeds up the learning. In particular, the perceptron learning, when combined with the less efficient message-passing MAP inference algorithm, is still shown to faster than maximum likelihood learning [24]. Now combined with our fast inference algorithm, the perceptron learning is able to speed up more.

3.4 Methods

We propose an inference algorithm that combines limited ‘truncation’, i.e. replacing a subset of non-regular potentials with regular approximations, with the QPBO method that does partial inference. This procedure resolve more variables than QPBO, and in some cases provably more correct assignments. We also propose a procedure that gives approximate assignment to unresolved variables if there are any left.

As we described earlier, the terms in the energy formulation Eq. (3.1) can be decomposed into node terms, regular pairwise terms, non-regular pairwise terms with
only one non-zero component, and triplet terms with only one non-zero component:

\[
\begin{align*}
\text{Energy}(\mathbf{X}) &= \sum_i N_i(X_i) + \sum_{i,j,k} T_{i,j,k}(X_i, X_j, X_k) + \sum_{i,j} E_{ij}(X_i, X_j) + \sum_{\{i,j\} \in M} F_{ij}(X_i, X_j) \\
&= \text{Regular}(\mathbf{X}) + \mathbf{F}(\mathbf{X})
\end{align*}
\]

where \(E_{ij}(X_i, X_j)\) is the regular pairwise term, \(F_{ij}(X_i, X_j)\) is the non-regular pairwise term with only one non-zero component as in Table 3.6: \(F_{ij}(0,0) > 0\), and \(M\) is the set of variable pairs in all non-regular pairwise terms.

The intuition is that, the fewer non-regular terms, the more variables we can resolve. If we drop all the non-regular terms, we can get a complete assignment. However, what is the relationship between the assignment we get from this ‘truncated’ MRF and the true MAP assignment? We have the following theorem:

**Theorem 1.** Assume \(\text{Energy}(\mathbf{X})\) consists of only node terms and pairwise terms. Given its truncation:

\[
\text{Energy}'(\mathbf{X}) = \text{Regular}(\mathbf{X})
\]

Assume \(x'^*\) is the MAP assignment to \(\text{Energy}'(\mathbf{X})\), which can be computed efficiently and exactly by mincut inference. There exists an assignment \(x^*\) that is the MAP assignment to \(\text{Energy}(\mathbf{X})\) and satisfies \(x^*_i = 1\) if \(x'^*_i = 1\).

The theorem follows the intuition that if we drop the non-regular terms, which disfavors the \((0,0)\) pair, we will end up with more 0’s in the MAP assignment and those assigned to be 1 are guaranteed to be correct.

**Proof.** Let \(\mathbf{Y}\) denote to be those variables in \(x'^*\) that are assigned to be 0 and \(\mathbf{Z}\) to be those variables that are assigned to be 1.

The MRF \(\text{Energy}'(\mathbf{X})\) can be converted into a mincut graph with a node \(v_i\) for each variable \(X_i\) plus two terminal vertices \(s\) and \(t\). The maxflow algorithm that solves the mincut problem pushes as much flow from \(s\) to \(t\) as possible. The mincut
in the residual graph\cite{74} has cost 0, i.e. there is no flow from the set $S$ to $T$. Pushing
the flow can also be viewed as reparametrization\cite{74}: if we convert the residual graph
back to an MRF (of only node terms and pairwise terms), it only differs from the
original MRF by a constant, which is the amount of the flow pushed. Therefore, we
have the following normal form:

$$
Energy'(y, z) = const + \sum N_i(y_i) + \sum M_i(z_i)
+ \sum E_{ij}(y_i, y_j) + \sum D_{ij}(y_i, z_j) + \sum C_{ij}(z_i, z_j)
$$

where the right side is converted back from the residual graph and it satisfies the
following properties because $y = 0, z = 1$ is a mincut of cost 0 for the residual graph,
i.e. there is 0 flow from $s \cup Y$ to $t \cup Z$:

- $y_i, y_j, z_i$, or $z_j$ is assignment to individual variable consistent with $y$ or $z$.
- $N_i(0) = 0$ and $N_i(1) \geq 0$.
- $M_i(1) = 0$ and $M_i(0) \geq 0$.
- $E_{ij}(0, 0) = E_{ij}(1, 1) = 0, E_{ij}(0, 1) \geq 0$, and $E_{ij}(1, 0) \geq 0$.
- $D_{ij}(0, 0) = D_{ij}(1, 1) = D_{ij}(0, 1) = 0$ and $C_{ij}(1, 0) \geq 0$.
- $C_{ij}(0, 0) = C_{ij}(1, 1) = 0, C_{ij}(0, 1) \geq 0$, and $C_{ij}(1, 0) \geq 0$.

Thus, we have:

$$
\begin{align*}
Energy(y, 1) \\
= const + \sum N_i(y_i) + \sum M_i(1) + \sum E_{ij}(y_i, y_j) + \sum D_{ij}(y_i, 1) + \sum C_{ij}(1, 1) \\
+ F(y \cup 1) \\
\leq const + \sum N_i(y_i) + \sum M_i(z_i) + \sum E_{ij}(y_i, y_j) + \sum D_{ij}(y_i, z_j) + \sum C_{ij}(z_i, z_j) \\
+ F(y \cup z) \\
= Energy(Y = y, Z = z)
\end{align*}
$$
The inequality are based on the inequalities for the corresponding terms. Most of them are obvious based on the properties of the normal form. For example, $D_{ij}(y_i, 1) \leq D_{ij}(y_i, z_j)$ because $D_{ij}(y_i, 1)$ is always 0 no matter what $y_i$ is, while $D_{ij}(y_i, z_j) \geq 0$. Finally, the inequality involving $F()$ is due to the $(0,0)$ pairs in $y \cup 1$ is a subset of those in $y \cup z$, and $(0,0)$ pairs are the only non-zero (positive) components in $F_{ij}()$, whose sum is $F()$.

We conclude there always exists a MAP assignment where we have $z = 1$.

In practice, we also consider the partial truncation: $Energy''(x) = \text{Regular}(x) + \sum_{\{i,j\} \in M_0} F_{ij}(x_i, x_j)$ where $M_0 \subset M$. We use the QPBO method to get a partial assignment to $Energy''()$. The intuition is that since the partially ‘truncated’ MRF disfavors the $(0,0)$ pair, our assignment of 1 is more conservative and is likely to be correct for the original $Energy()$.

Therefore, our algorithm starts with $\text{Regular}(x)$. At each step, it incrementally adds in one non-regular term $F_{ij}()$. We apply the QPBO method on this partial truncation and produce the partial assignment. We repeat the steps until all non-regular terms are added. We pick among all the partial assignments the one that has the most variables assigned to be 1. We fix those variables to be 1, and repeat the same procedure on the remaining variables at the next iteration. We are guaranteed to fix the correct variables if we pick the partial assignment corresponding to the complete truncation, $\text{Regular}(X)$ at the beginning of the iteration, due to our Theorem 1. On the other hand, it is also guaranteed to be correct if we pick the partial assignment corresponding to the original MRF at the end of the iteration when all non-regular pairwise terms are added due to the correctness of the QPBO algorithm. Note that the 0 and 1 labels are symmetrical. Therefore at each iteration, we can also decompose the non-regular terms so that $F_{ij}(1,1) > 0$ is the only non-zero component, and conservatively fix some variables to be 0 at each iteration. We repeat the iterations until no more variables could be fixed.

During each iteration, we need to run mincut multiple times, each time with one more non-regular term. Using the dynamic graph cuts algorithm by Kohli and
Torr [71], the time complexity of repeatedly running the mincut for each of the non-regular terms within a certain iteration is on the same level of magnitude as running one single mincut on the original energy function. Therefore, our method is still guaranteed to be fast.

If there are still unresolved variables at the end of the algorithm, we use an approximation method to complete their assignment. In the mincut graph constructed by the QPBO method, the two vertices for an unresolved variable, $p$ and $\bar{p}$, are in the same strongly connected component [74]. We compare the maximum amount of flow we can push from $p$ to $\bar{p}$ to the amount we can push from $\bar{p}$ to $p$. This can be shown to be equivalent to the procedure used by Kohli and Torr [72] to compute the max-marginals, but extended to this non-regular case. In the case when all variables are resolved, the amount of flow pushed is proportional to the max-marginal. Therefore if we can push more flow from $p$ to $\bar{p}$, the corresponding variable would have a higher max-marginal when assigned to 1 than when it is assigned to 0. In the case involving unresolved variables, it is no longer guaranteed to be true, but we still use it as an approximation. Thus, here is our procedure for assigning an unresolved variable: if we can push more flow from $p$ to $\bar{p}$ than from $\bar{p}$ to $p$, we assign the corresponding variable to 1; otherwise, we assign it to 0. We fix this variable and repeat the procedure for the next unresolved variable. We can do this for all unresolved variables efficiently using the dynamic graph cuts algorithm [71].

To summarize, we proposed a fast inference algorithm that works on any kind of MRF. We applied it to the protein-protein interaction data, which we will describe in details below. We also applied it to object recognition in computer vision. It is shown to have comparable accuracy to loopy belief propagation, while being almost 10 times faster.
3.5 MRF for the triplet model

3.5.1 Representation

We extend the model of Jaimovich et al. [64]. We have four types of nodes in the MRF for predicting protein-protein interactions:

- Actual protein-protein interactions $I_{ij}$. They have value 1 if protein $i$ and protein $j$ interact and 0 otherwise.

- Observed protein-protein interactions $IA_{ij}$. They have value 1 if protein $i$ and protein $j$ are observed to interact in a particular assay and 0 otherwise. We used four interaction assays: Ito, Uetz, DIP, and MIPS and thus have four IA nodes for each protein pair.

- Actual protein localization $L_i$. They have value 1 if protein $i$ locates in the particular cellular component, and 0 otherwise. We use GFP experimental data, which distinguishes between four different cellular components: nucleus, cytoplasm, mitochondrion, and endoplasmic reticulum [61] so we have four nodes for each protein.

- Transcriptional regulation node $R_{ij}$. $R_{ij}$ is 1 if protein $i$ transcriptionally regulates protein $j$, and 0 otherwise. We use data from ChIP-Chip experiments by [85] as the actual regulation.

During the training, all nodes are observed. During test, we do inference on the actual protein-protein interaction nodes given all the other nodes.

The MRF has four types of cliques:

- Directed edges between actual protein-protein interactions $I_{ij}$ and observed protein-protein interactions $IA_{ij}$ to deal with the noise in the protein interaction assays.

- Triplets between actual protein-protein interactions $I_{ij}$ and their corresponding actual localizations $L_i$ and $L_j$ to represent the intuition that interacting proteins should be co-localized.
• Triplets for three actual protein-protein interaction nodes: $I_{ij}$, $I_{jk}$, and $I_{ik}$ to represent the transitivity relationship between them.

• Triplets for $R_{ij}$, $R_{ik}$, and $I_{jk}$ to represent the intuition that if two proteins are co-regulated by the same other protein, they are more likely to interact with each other.

3.5.2 Learning and inference

We maximize the joint likelihood of the data using gradient ascent. We tried two training methods: generative where we maximize $P(I, IA, L, R)$ and discriminative where we maximize $P(I|IA, L, R)$. We used the MAP assignment computed by the efficient mincut inference to approximate the gradient.

The MAP assignment for $P(I, IA, L, R)$ in the generative case is:

$$\arg\max_{I, IA, L, R} P(I, IA, L, R)$$

where $IA$, $L$, and $R$ are not fixed.

The MAP assignment for $P(I|IA, L, R)$ in the discriminative case is

$$(I, IA, L, R) \text{ s.t. } I = \arg\max_{I} P(I|IA, L, R)$$

where $IA$, $L$, and $R$ are fixed to the observed values.

We use the exact mincut inference during test time because we want to apply the learned model to entire proteome and belief propagation is too slow for that. Mincut inference, though efficient and exact, requires the potentials to be regular. Therefore, at each step of the gradient ascent during the learning, we use the log-barrier method [16] to force the parameters to stay in the regularity constraints. Although this limits the class of potentials we can learn, it speeds up both inference and learning, which uses MAP inference as a subroutine to compute the approximate gradient. Since our potentials are close enough to be regular, the approximation is accurate enough to generate good results.
3.5.3 Experiment setup

We evaluated our model on a S. cerevisiae data set compiled by von Mering et al. [130] who ranked 80,000 protein-protein interactions by their reliability. We picked the top 1,000 interactions and considered them as the ‘true’ interactions. Those 1000 interactions involve 543 unique proteins. From those proteins, we generated the ‘true’ non-interactions by randomly selecting 10,000 protein pairs that do not appear on the list of 80,000 interactions. We used 10 times as many non-interactions as interactions so we have roughly the same number of triplets involving non-interactions as triplets involving interactions. This avoids the intrinsic bias in the model of Jaimovich et al. Section 3.2. We used four interaction assays [127, 63, 98, 122] and one localization assay with four cellular components [61]. We also used transcription regulation data from [85], which resulted in 113 transcription factors each regulating 39 target genes on average. We used standard four-fold cross validation, where we learn the model using 750 interactions and 7,500 non-interactions and test it on the remaining held-out protein pairs.

3.5.4 Results

We compare our model with the two best-performing models used by [64]. The so-called ‘Triplet’ model includes both the interaction and localization variables and cliques. As such, it is a strict subset of the ‘Regulation’ model described earlier. The so-called ‘Full’ model extends the ‘Triplet’ model by hiding the actual localization variables $L_i$, and adding respective observed variables $LA_i$ corresponding to the experimental GFP results. The hidden variable is handled by EM during the learning, where we iteratively assign values to the $L_i$’s given the current parameters and then optimize the parameters based on the complete assignment.

As we can see from Fig. 3.2, in the generative learning regime originally used by Jaimovich et al. [64], we are able to improve the accuracy of the ‘Triplet’ model by integrating the regulation data and reach accuracy comparable to the ‘Full’ model. Moreover, our discriminative learning regime yields significantly improved cross-validation performance for all models.
Figure 3.2: Sensitivity of the three MRF models in predicting protein-protein interactions. We cut off our predictions and compute the sensitivity at a level where we achieve 99.5% specificity, where sensitivity is the proportion of labeled positives that we actually predicted to be positive, and specificity is the proportion of labeled negatives that we actually predicted to be negative. The ‘Regulation’ model is our extension to the ‘Triplet’ model and the ‘Full’ model is by [64]. The red bar is for generative training where we maximize the likelihood all data while the blue bar is for discriminative training where we maximize the likelihood of the labels given other observations.
Figure 3.3: **Computational time for learning three MRF models.** Computational time for applying our learning algorithm, which is based on efficient MAP inference, to the three MRF models are shown in red and blue for generative and discriminative training respectively. In comparison, computational time for the original maximum likelihood learning algorithm by Jaimovich *et al.* [64], which is based on loopy belief propagation inference, is shown in green.
As shown in Fig. 3.3, our learning algorithm, which is based on efficient MAP inference, reduces the training time of the two original models significantly compared to that based on loopy belief propagation by Jaimovich et al. [64]. It allows us to learn our new ‘Regulation’ model with thousands of co-regulation triplets efficiently. Moreover, efficient inference would enable us to expand our MRF to cover more proteins in the genome.

3.6 MRF for the complex model

3.6.1 Representation

To construct complexes, we try to decide for each protein \( j \), whether it belongs to a particular complex \( i \) or not. We build an MRF to represent the relationship between proteins and complexes. For each protein \( j \) and complex \( i \), we create a node \( V_{ji} \), whose value 1 if protein \( j \) is in complex \( i \) and 0 if protein \( j \) is not in complex \( i \). Then we associate two types of cliques with the Markov Network:

- Each node has a singleton clique with a potential function that is \( b \) when the node value is 1 and 0 when the node value is 0. This node potential represents the prior probability that a protein is in a particular complex.

- For any two nodes for the same complex \( i \), \( V_{ij} \) and \( V_{ik} \), we create an edge between them: \( E_{ijk} \). The potential for this edge is \( w^Tf \) if both nodes have values 1, and is 0 otherwise, where \( f \) are the features between protein \( i \) and protein \( j \) such as TAP-MS score and co-localization. See next chapter for a complete list of features we use. \( w \) is the vector of weights we need to learn; it weighs the different feature appropriately so \( w^Tf \), which is the affinity between the two proteins, corresponds to the likelihood the two proteins being in the same complex. Ideally, \( w^Tf \) should be large for a pair of proteins in the same complex, and small otherwise. Our learning tries to find the \( b \) and weights \( w \) that best explains our reference complexes. Using the learned model, our inference tries to identify new complexes that have high likelihood, i.e. the sum of affinities between all pairs of proteins in the complex.
3.6.2 Learning and identifying complexes

We use the same training and test set as in the next chapter (Section 4.3 and Section 4.6). We ended up having 340 reference complexes, four-fifth of which are used during training in a five-fold cross validation. We maximize the likelihood of the MRF constructed from the training set. All nodes in the MRF are labeled. The gradient for the maximum likelihood optimization is approximately computed using belief propagation.

To identify a new complex $C$, we construct an MRF that for each protein, it has a node that associates the protein with the complex $C$. We connected all pairs of nodes into pairwise cliques as described earlier. We then do inference in the MRF using the learned weights. However, the potentials for some edges would be non-regular because, for a pair of proteins not likely to be in the same complex, we would have a negative value of $w^T f$, which means the 1-1 configuration is less likely between the nodes representing the pair of proteins. To deal with such an MRF with non-regular potentials, we use our efficient inference algorithm described earlier, which is shown to work well empirically in the object recognition in computer vision. The resulting assignment to the nodes tells us whether the proteins belong to the new complex $C$.

In order to avoid repeatedly identifying the same complex, we find two proteins, $P_i$ and $P_j$, not in any of the same complex discovered so far and fix their value to be 1. Therefore, the new complex would always include $P_i$ and $P_j$ and thus be different from the complexes already discovered. However, the new complex may involve proteins that have high affinity with each other but have low affinity with $P_i$ or $P_j$, which are included because they are fixed. Therefore in a post-processing stage, we run the inference a second time only on the proteins in the complex discovered in the previous stage, but this time we do not fix $P_i$ and $P_j$. The complex resulting after the post-processing is considered to be our predicted complex, which may be redundant to our previously predicted complexes.

3.6.3 Experiment setup

Refer to the experiment setup in the next chapter (Section 4.6).
CHAPTER 3. MRFS: MODELING INTERACTION AND COMPLEX

Figure 3.4: Verification of complex predictions using MRF.
We use five-fold cross-validation to predict reference complexes in the held-out set that is not used in training. The blue bar shows the reference complex that exactly matches certain predicted complex. The green and red bars shows the reference complex that differs with certain predicted complex by one and two proteins respectively. We collectively call those reference complexes well-predicted. See next chapter for exact definition. We compare our MRF model with the state-of-the-art methods of Hart et al. and Pu et al.

(a) The total number of reference complexes that are well predicted.
(b) The number of reference complexes well predicted divided by number of predicted complexes. This corresponds to the sensitivity of the predictions. Our MRF model is able to achieve higher sensitivity with fewer predicted complexes.

3.6.4 Results
We constructed a reference set of reliable complexes. See next chapter for details on how we construct the reference complexes. We did five-fold cross-validation to evaluate the accuracy of our complex reconstruction.

We compare our method with the state-of-the-art methods of Hart et al. [57] and Pu et al. [109]. As we can see from Fig. 3.4(a), Hart et al. and Pu et al. are able to well predict more reference complexes than our MRF model. However, this is mostly due to the fact that our method predicts much fewer complexes (182) than Hart et al. (337) and Pu et al. (396). For each predicted complexes, our method is able to perfectly predict 0.264 reference complex on average, compared to 0.199 and 0.164 reference complex by Hart et al. and Pu et al. respectively (Fig. 3.4(b)). If we
define the sensitivity of the predictions to be the number of reference complexes well predicted divided by the number of predicted complex, our method is able to achieve higher sensitivity than Hart et al. and Pu et al..

A shortcoming of our method is that it predicts fewer complexes because many of the complexes it found are redundant. Therefore, fewer total number of reference complexes are well predicted Fig. 3.4(a), resulting in low coverage of our method. We will present new methods in the next chapter to expand the coverage of our predictions.

3.7 Discussion

There are many work on predicting protein-protein interactions. Some of them [14] use flat models where predictions on protein-protein interactions are made independently. Others, such as our InSite model of the last chapter, use a graphical model so the interaction predictions are correlated indirectly through the affinities between motifs.

In this chapter, we extend the model of Jaimovich et al. [64]. It takes into account the transitivity relationship among protein-protein interactions. Therefore, predictions in one part of the protein-protein interaction network give signals to predictions in another part of the interaction network, which in turn give signals to predictions in yet another part of the network. We use the framework of MRF to encode such transitivity relationship and make predictions on all protein-protein interactions at the same time using an efficient inference algorithm we developed. With the flexible framework of MRF and our efficient inference algorithm, we also encode the relationships that interacting proteins are more likely to be located in the same cellular component and they are more likely to be regulated by the same transcription factors. Our results show that we are able to better predict protein-protein interactions with those additional features.

There are many other types of relationships between proteins that are related to protein-protein interactions [142]. For example, interacting proteins are more likely to be phosphorylated by the same kinase. With our MRF framework, it is easy to
encode those additional relationships. In the case we do not know, a priori, whether certain relationship exists, we can still put it into our MRF and use $L_1$ regularization to do feature selection and remove those relationships that do not exist [82].

Many protein-protein interactions and their transitivity relationship are the results of multiple proteins in the same complex. In the second part of this chapter, we try to reconstruct complexes directly. Our MRF model picks a set of proteins that has the highest sum of affinity between the member proteins. Therefore, it tends to pick the same set at different iterations, resulting in complexes that are exactly the same as each other. After removing this redundancy, we ended up with few predicted complexes. Therefore, our model has low coverage despite its high sensitivity. We address this problem in the next chapter with methods based on supervised learning and clustering. Another solution would be to reduce the affinities between proteins that already appear in any of the same complex, as Gavin et al. [44] did. This way, it is less likely to predict the same complex again because the affinities within that complex are reduced.
Chapter 4

Stoichiometrically Stable Complexes

4.1 Introduction

Biological processes exhibit a hierarchical structure in which the basic working units, proteins, physically associate to form stoichiometrically stable complexes. Complexes interact with individual proteins or other complexes to form functional modules and pathways that carry out most cellular processes. Such higher level interactions are more transient than those within complexes and are highly dependent on temporal and spatial context. The function of each protein or complex depends on its interaction partners. Therefore, a faithful reconstruction of the entire set of complexes in the cell is essential to identifying the function of individual proteins and complexes, as well as serving as a building block for understanding the higher level organization of the cell, such as the interactions of complexes and proteins within cellular pathways. In this chapter, we describe a novel method for reconstruction of complexes from a variety of biological assays.

Our reconstruction effort focuses on the yeast Saccharomyces cerevisiae, both as the prototypical case study for the reconstruction of protein-protein interaction networks; importantly, the yeast complexes often have conserved orthologs in other organisms, including human, and are therefore of interest in their own right. Several
studies [45, 59, 44, 79], using a variety of assays, have generated high-throughput data that directly measure protein-protein interactions. Most notably, two high-quality data sets [44, 79] used tandem affinity purification followed by mass-spectrometry (TAP-MS) to provide a proteome-wide measurement of protein complexes. These data provide the basis for attempting a comprehensive reconstruction of a large fraction of the protein complexes in this organism.

Despite the fairly high quality of these networks and the agreement between them, they still contain many false positives and negatives. False negatives can arise, for example, from the difficulty in detecting interactions involving low-abundance proteins or membrane proteins; or from cases where the tag added to the bait protein during TAP-MS prevents binding of the bait to its interacting partners. False positives can arise, for example, from complexes that share components; or from the contaminants that bind to the bait non-specifically. Therefore, the set of complexes derived from the protein-protein interaction network alone has limited accuracy. Less than 20% of the MIPS complexes [97], which are derived from reliable small-scale experiments, are exactly captured by the predictions of Pu et al. [109] or by those of Hart et al. [57].

In this chapter, we construct a method that generates a set of complexes with higher sensitivity and coverage by integrating multiple sources of data, including mRNA gene expression data, cellular localization, and yeast 2-hybrid data. These evidences, however, only provide weak signals to co-complexness and they also correlate with other relationship between two proteins such as being in the same pathway. Therefore, we develop a data integration approach that is aimed directly at the problem of predicting stoichiometrically stable complexes.

In the previous chapter, we used an MRF model to identify complexes, which has high sensitivity but low coverage. It is equivalent to finding the subgraph that has the maximum sum of affinities between all the pairs in the subgraph, where the affinity is the result of combining the learned weights with the features between a pair of proteins. To avoid this restricted form of defining cluster coherence, we try new models that learn a cluster coherence measure directly from raw evidences, such as TAP-MS score, instead of collapsing them into affinities for pairs of proteins first. This allows us to use richer features across protein pairs to define the cluster.
coherence, and a more flexible way to combine the affinities for all protein pairs.

We began by creating a comprehensive set of reference complexes from the literature. Unlike other methods, which generally used only the MIPS [97] complexes, we extracted complexes from both MIPS (225 complexes) and SGD [23] (195 complexes), and combined them with a large set of (164) hand-curated complexes constructed from our own prior knowledge. We then applied an unbiased procedure to unify these (sometimes inconsistent) sets into a large set of 340 reference complexes that we used both for training our learning method and for evaluating the quality of its predictions (in a hold-out regime). The merging process creates complexes that are supported by multiple sources, and whose protein members appear in strict majority of the sources. Therefore, the resulting reference set has both higher sensitivity and coverage than those used by previous studies [57, 109]. This set of high-quality reference complexes can be downloaded from our website [4].

Based on this set, we tried three different algorithms: a complexness model, a protein-complex model, and a protein-protein model.

The complexness model tries to learn a ranking between two sets of proteins based on the features between all pairs of proteins in each set. The ranking tells us which set of proteins looks more like a complex. We identify complexes by doing greed hill climbing to find a set of proteins that is a local maximum of the ranking function.

The protein-complex model tries to learn a classification that decides whether a protein should belong to the same complex with a set of proteins based on the features between the protein and every protein in the set. Starting with a set of proteins, we then use a procedure that iteratively adds proteins with positive score to the set and removes an existing protein from the set if it has negative score.

The protein-protein model works the best and is the focus of this chapter. It, like others, has two phases. In the first, we use boosting [24], a state-of-the-art machine learning method, to train an affinity function that is specifically aimed at predicting whether two proteins are co-complexed. Unlike most other learning methods, boosting is capable of inducing useful features by combining different aspects of the raw data, making it particularly well-suited to a data-integration setting. Once we generate the learned affinity graph over pairs of proteins, we predict complexes by using a novel
clustering algorithm. Our initial experiments showed that hierarchical agglomerative clustering (HAC), which progressively merges sets of proteins with strongest affinity, produces the best results for complex reconstruction if trained to optimize for that task. However, HAC has several significant limitations. First, it does not allow clusters to overlap, whereas actual complexes do share subunits. Second, it uses a single cutoff to decide the granularity of the complexes constructed. A cluster near the cutoff in the dendrogram can be formed even if it is the result of merging two relatively weakly connected sub-clusters A and B. Such a cluster, although of lower confidence, still excludes both A and B from being predicted as a complex; this occurs even if A and B are strong candidates for being a complex. Finally, once a set of proteins is merged with another set, it cannot merge with anything else even if the affinity is only slightly lower. Therefore an incorrect decision cannot be fixed later in the process.

To address these limitations, we constructed a novel clustering algorithm called HACO (HAC with Overlap) that allows a set of proteins to be merged with multiple other sets with which it has comparably strong affinity. HACO addresses all of the limitations above: It produces clusters that can overlap. Second, when merging A and B into a single cluster C, it also has the option of leaving A and/or B as candidate complexes, avoiding a wrong decision because of an arbitrary cutoff. Finally, as it allows the same cluster to be used in multiple places, it avoids many mistakes that arise from an almost-arbitrary breaking of near-ties. Both our boosting algorithm and the HACO code are made freely available on our project webpage [4], allowing them to be used for predicting complexes with other forms of data. Moreover, the HACO algorithm is a simple and elegant extension of HAC, which can be applied to any setting where HAC is applied; given the enormous usefulness of HAC for the analysis of biological data sets of many different types (e.g., [34]), we believe that HACO may be applicable in a broad range of other tasks.

To validate our approaches, we show that we are able to predict more reference complexes in the held-out set that is not used in training. By integrating multiple sources of data, we recover more reference complexes than other state-of-the-art methods [57, 109], even when we use simple HAC for the clustering. We further
improve both the coverage and sensitivity of our predictions when we use HACO. In particular, Hart et al. [57] and Pu et al. [109] are only able to predict 67 and 65 complexes respectively that exactly matches some reference complex. On the other hand, HACO is able to predict 95 complexes perfectly. We also validated our predicted set of complexes against external data sources that are not used in the training. In all cases, our predictions are shown to be more coherent than methods of Hart et al. and Pu et al.. Interestingly in two of the four cases, our predictions are even more coherent than the reference set of complexes: proteins in the same predicted complex share more transcription regulators and they have similar abundance levels.

Our predicted set of complexes provides us with some new insight on the global structure of the protein complex network. In the past, Jeong et al. [66] have suggested that the degree of a protein in an interaction network is positively correlated with its essentiality, and have argued that ‘hubs’ in the network are more likely to be essential because they are involved in more interactions. Our analysis shows that this phenomenon is much better understood once we understand the protein network in terms of complexes. Hart et al. [57] recently showed that complexes are either ‘essential’ — have a large fraction of essential components — or inessential — having a small fraction of such components. We show here that large complexes are preferentially comprised of essential proteins: the larger the complex, the larger the fraction of essential proteins. Indeed, the size of the (largest) complex to which a protein belongs is a significantly better predictor of its essentiality than its overall network connectivity.

4.2 Related work

A number of works [57, 109] have attempted a comprehensive reconstruction of a large fraction of the protein complexes in Yeast. Generally speaking, all use the same general procedure: one or more data sources are used to estimate a set of affinities between pairs of proteins, essentially measuring the likelihood of that pair to participate together in a complex; these affinities induce a weighted graph, whose nodes are proteins and whose edges encode the affinities; a clustering algorithm is
then used to construct complexes C sets of proteins that have high affinity in the graph. Although similar at a high level, the different methods differ significantly on the design choices made for the key steps in the process.

Recent works (since 2006) all focus on processing the proteome-wide TAP-MS data and using the results to define complexes. Gavin \textit{et al.} [44], Collins \textit{et al.} [25], and Hart \textit{et al.} [57] all use probabilistic models that compare the number of interactions observed between proteins in the data versus the number expected in some null model. Collins \textit{et al.} and Hart \textit{et al.} both used all three of the available high-throughput data sets [59, 44, 79], in an attempt to provide a unified interaction network. The two unified networks resulting from these studies were shown to have large overlap and to achieve comparable agreement with the set of co-complex interactions in the MIPS data set [97], which are collated from previous small-scale studies. The interaction graphs resulting from the computed affinity scores are then clustered to produce a set of identified complexes. Gavin \textit{et al.} [44], Hart \textit{et al.} [57], and Pu \textit{et al.} [109] all use a Markov clustering [36] (MCL) procedure; Collins \textit{et al.} [25] use a hierarchical agglomerative clustering (HAC) procedure, but do not suggest a computational procedure for using the resulting clustergram to produce specific complex predictions. Following are the details of several complex reconstruction methods.

- Bader \textit{et al.} [10] used a novel clustering algorithm called Molecular Complex Detection (MCODE) to detect densely connected regions in an earlier data set of protein-protein interactions. It starts by assigning a weight to each protein based on its neighborhood density. Then it picks the top-weighted protein as a seed and traverse out from it to include neighboring proteins whose weights are above a threshold. Once it stops, all the proteins picked along the way form a complex, which is then excluded from the network in the following rounds that start from the next highest weighted protein. However, Brohee \textit{et al.} [18] showed that the Markov clustering algorithm (MCL) works better than MCODE on protein-protein interaction network.

- Gavin \textit{et al.} [44] first computed a socio-affinity score between each pair of proteins to be the log-odds of the number of times the two proteins are observed
together in some purifications relative to what is expected by chance based on their frequencies. It takes into account both bait-prey and prey-prey information and is unbiased toward known complexes. The pairwise network of socio-affinity scores is then subjected to a procedure that produces overlapping clusters. This clustering procedure is repeatedly performed with different parameters. Similar clusters resulted from different parameters are grouped to form ‘complex isoforms’. Proteins in each complex are divided into core, which appears in most of the isoforms, and attachment, which only appears in some of them. Two or more proteins in some attachment that also appear in other complexes comprise a module.

- Krogan et al. [79] used a machine learning approach, trained on MIPS reference complexes, to predict the confidence score for a pair of proteins to be in the same complex. It only uses bait-prey relationship. It then applied a Markov Clustering (MCL) algorithm to the pairwise network of confidence scores to produce a list of non-overlapping clusters.

- Hart et al. [57] defined a p-value by comparing observed relative to expected number of interactions in both bait-prey and prey-prey relationship. It is applied to three sets of purifications [59, 44, 79] and the combined score is derived from multiplying their p-values. It then applied MCL to produce a list of non-overlapping clusters.

- Pu et al. [109] applied MCL directly to the purification enrichment (PE) score [25], which is derived from two sets of purifications [44, 79]. The clusters from MCL were post-processed to identify proteins that are likely to be recruited by multiple complexes. This resulted in a list of overlapping complexes. Pu et al. [109] showed using PE score to combine Gavin’s and Krogan’s purifications gets better accuracy than using either purification alone and MCL produces state-of-the-art complexes.

- Collins et al. [25] applied a hierarchical agglomerative clustering (HAC) procedure to the PE score. However, they do not suggest a computational procedure
for using the resulting clustergram to produce specific complex predictions. Instead, biologists look for potential complexes as regions of dense connections in the clustergram.

Unlike the above methods, we generate a set of complexes with higher sensitivity and coverage by integrating multiple sources of data, including mRNA gene expression data, cellular localization, and yeast 2-hybrid data. The data integration approach was used in some early works on predicting protein-protein interactions [65, 143], but was not revisited in recent years. They tried, however, only to predict whether two proteins belong to the same complex without reconstructing the set of proteins that constitutes a complex. Jansen et al. [65] used a full Bayesian Network to integrate four different high-throughput experiments of protein-protein interactions. The complexity of a full Bayesian Network grows exponentially with the number of features, and thus it is unlikely to be extended to deal with more features. Zhang et al. [143] used a probabilistic decision tree to decide whether a protein pair belongs to the same complex or not. A decision tree, though easily interpretable by biologists, fragments the data with the addition of each layer of the tree.

Many recent studies [22, 84, 92, 117, 124, 129, 131, 135, 138] have successfully integrated multiple types of data to predict functional linkage between proteins, constructing a graph whose pairwise affinity score summarizes the information from different sources of data. In particular, Chen and Yuan [22] integrated protein-protein interactions and expression data to build a weighted graph. The resulting clusters, however, are functional modules, which are larger units and sometimes are supersets of complexes. Lee et al. [84] and Marcotte et al. [92] integrates multiple sources of genomic data, such as genetic interactions, co-evolution, co-expression, and domain fusion to predict pairwise functional relationship, which is used to assign protein functions.

However, since the data integration is not trained toward predicting complexes, the high-affinity pairs contain transient binding partners, and even protein pairs that never interact directly but merely function in the same pathways. When these graphs are clustered, the clusters correspond to a variety of cellular entities, including pathways, functional modules, or co-expression clusters. We develop a data integration
approach that is aimed directly at the problem of predicting stoichiometrically stable complexes.

4.3 Constructing a set of reference complexes

We compiled a reference set of complexes by combining literature-derived results from small-scale experiments in MIPS [97] and SGD [23] with a hand-curated list (see our supporting website [4]) that we generated. The MIPS, SGD, and hand-curated set contain 225, 195, and 164 complexes respectively. Below we describe our method for establishing correspondence between the three lists and combining them into a high-confidence reference set suitable for training our method and for evaluating the accuracy of its predictions.

Our approach consisted of five processing steps. First, we merged similar complexes from the original lists (see below), resulting in a list of 543 complexes. Second, we removed 112 redundant complexes which were proper subsets of other complexes. Third, we removed the five largest complexes: the four ribosomal subunits and the small nucleolar ribonucleoprotein complex; these complexes are so large that they greatly overwhelm the signal, both in training the method and in evaluating the results. Fourth, we restricted the complexes to the set of 2195 proteins that have adequate amount of experimental evidence (see below). Finally, we removed single-protein complexes, arriving at the final list of 340 complexes. With at least 2 and on average 4.9 proteins per complex, this set of complexes contained 1100 unique proteins and a total of 1661 protein members, showing that the reference complexes contain notable overlap (proteins that are shared by multiple complexes).

In the first step of this merging process, we define each candidate complex from the three curated lists as a node in an undirected graph (or network). Two complexes are connected by an edge if they overlap significantly, i.e., their Jaccard coefficient is greater than 0.7 (see JC metric below), with an edge weight equal to the JC value. We found 422 isolated nodes in the graph, corresponding to unique complexes that do not overlap significantly with any other complexes in the list. The task of merging similar complexes is equivalent to that of finding several types of connected components in
this graph. A complete subgraph with average edge weight of 1 is equivalent to a
group of complexes with identical protein content that appear under multiple names
in at least two of the curated lists. We found 66 such groups, which correspond to
complexes that we regard as very high-confidence because of multiple corroborating
evidence. A complete subgraph in the rest of the network with average edge weight
less than 1 (but greater than 0.7) is equivalent to a group of complexes whose protein
contents are reported differently by the different curated lists. We found 45 such
groups and produced a consensus complex for each, resolving conflicts by a majority
vote: a protein was included in the resulting complex only if it was found in more
than half of the candidate complexes from the conflicted group. The remaining 18
nodes formed 4 connected components but no complete subgraphs, each component
indicating non-transitive overlaps between three or more candidate complexes (e.g.
A overlaps with B, and B overlaps with C, but A does not overlap significantly with
C). Manual inspection and consultation with experts resulted in 10 unique complexes
being added to the reference list.

4.4 Pairwise signals for predicting complexes

We extracted pairwise signals from five different data sources: the purification en-
richment (PE) score from the consolidated network of Collins et al. [25], a cellular
component from a truncated version of the Gene Ontology (GO) [9], trans-membrane
proteins [23], co-expression [49], and yeast two-hybrid (Y2H) interactions [63, 127].

Our highest-coverage source regarding direct physical interaction comes from hig-
throughput TAP-MS data of the Gavin [44] and Krogan [79] data sets. The recent
work of Collins et al. [25] provides a coherent and systematic way of integrating the
data from these separate assays into a high-quality score that measures the probability
of a protein pair to be co-complexed. The recent work of Hart et al. [57] provides
a different integration method, but the results are quite similar, providing support
for both of these procedures. We derived five signals from the PE analysis: the
direct score is computed based only on bait-prey information in the purifications; the
indirect score is computed based on prey-prey information; the actual PE score is
the sum of direct and indirect scores; the scaled score maps the PE score to a value between 0 and 1 to approximate the confidence value that the pair represents a true interaction; finally each protein is represented by a vector of its scaled PE scores with all the other proteins (where we assign its interaction with itself a score of 1), and we define our PE-distance signal as the cosine distance between the vectors of two proteins.

As the PE score provides most of the signals in predicting complexes (See Results section), we only kept the 2390 proteins that have at least one scaled PE score above 0.2 with some other protein. Although this set only covers about 40% of the approximately 6000 yeast genes, it covers 81% of all protein members in the lists of high-quality complexes that comprised our reference set. As noted earlier, we exclude proteins that appear exclusively in the four ribosomal subunits and the small nucleolar ribonucleoprotein complex. This resulted in the final list of 2195 proteins, on which we performed our complex prediction.

The Gene Ontology (GO) cellular component hierarchy [9] was downloaded on June 25, 2007. An examination of the hierarchy showed that many of the smaller categories (lower in the hierarchy) refer to particular complexes whose information is derived from the same small-scale experiment that inform our reference set. Thus, in order to achieve a fair evaluation using the reference set, we remove categories of size less than 120 that can potentially contain the answer. The remaining 44 out of 564 categories represent high-level cellular localization information, much of which is obtained through high-throughput experiments [61]. Some sample categories include ‘endoplasmic reticulum part’, ‘nuclear chromosome part’, ‘mitochondrial membrane’, and ‘cytoplasm’.

We derived two pairwise localization signals from the GO cellular component. One is the semantic distance measure [89], which is the log size of the smallest category that contains both proteins. However, this signal is a pessimistic assessment regarding the co-localization of the two proteins, as lack of annotation of a protein in some category, particularly one that is a subset of its most specific category, does not necessarily mean that it cannot belong to this category. Therefore, we construct a second signal, which is the log size of the smallest possible group that could contain both proteins.
(given the current evidence). It is computed in the following way between protein \( A \) and protein \( B \), whose most specific categories are \( X \) and \( Y \) respectively. If \( X \) is a sub-category of \( Y \), then the two proteins might belong together to any group if they were to be annotated with enough detail. Therefore, we use \( \log(120) \), the size of the smallest category, as our second signal. On the other hand, if \( X \) and \( Y \) are not sub-categories of each other, we denote \( Z \) to be the smallest common super-category of \( X \) and \( Y \). We then denote \( X' \) (resp. \( Y' \)) to be the category one level down the path from \( Z \) to \( X \) (resp. \( Y \)). Thus, assuming that \( A \) and \( B \) belong to the two different categories at \( X' \) and \( Y' \), the smallest semantic category that we can form that may contain them both is \( X' \cup Y' \). Thus, our second signal is \( \log(|X' \cup Y'|) \).

A list of membrane proteins are obtained by parsing the trans-membrane annotations in SGD [23]. A pair of proteins is considered to be membrane if at least one of the proteins is found in the membrane. The first membrane signal is 1 if the pair is membrane and 0 otherwise. The second and third signals are the product of the first signal with the direct and indirect PE score of the two proteins, respectively. This allows our boosting model to take into account the known fact that TAP-MS purifications work differently on membrane proteins from non-membrane proteins.

Yeast two-hybrid protein-protein interactions are obtained from the assays of Ito et al. [63] and Uetz et al. [127]. Interacting pairs are assigned signal value 1. Pairs of proteins that appeared in the assay but not observed to interact are assigned signal value -1. All other pairs have 0 as their signal values.

Microarray data were downloaded from Stanford Microarray Database (SMD) [49] on Dec. 5th, 2006, which contains a total of 902 experiments for Yeast divided into 19 categories. The data were normalized to mean 0 and standard deviation 1. We construct a signal by computing the mean-centered Pearson correlation coefficient between the expression profiles of two proteins.

A final signal is obtained from small-scale physical interactions. We downloaded protein-protein interactions from MIPS [96] and DIP [134] on 21 March 2006. We extracted from MIPS those physical interactions that are non-high-throughput yeast two-hybrid or affinity chromatography. For DIP, we picked non-genetic interactions that are derived from small-scale experiments or verified by multiple experiments.
This signal has value 1 for observed interactions, and signal value 0 for all other pairs. Importantly, there is a risk of cyclicity between these small-scale interactions and the reference complexes. Therefore, to avoid a positive bias in our results, we omitted this signal in the cross-validation runs, which are evaluated against the reference complexes. For those runs that are trained on the entire set of reference complexes, this cyclicity is not a concern, so this signal was included.

There are a total of 12 signals for cross-validation runs and 13 signals for runs that are trained on the entire reference set.

\section{4.5 Methods}

\subsection{4.5.1 Complexness model}

In the complexness model, we try to learn the coherence for a set of proteins, which measures how the set is like a complex.

\textbf{Feature construction}

Denote $S_j(P,Q)$ to be the $j$th signal between protein $P$ and protein $Q$. For each $j$, we construct features for a set of proteins $C$ by aggregating signals between proteins in the set:

$$f_{ij}(C) = A_i(\{S_j(P,Q)|P \in C, Q \in C, P \prec Q\})$$

where $A_i$ is the $i$th aggregation function that combines the $j$th signals for all pairs of proteins in the set. $\prec$ can be any ordering among the proteins, such as alphabetical, to avoid a pair appearing twice in both orders. We used the list of aggregating functions in Appendix A.

Besides the pairwise data, we also use the GFP localization data, which give us the localization of a protein in five cellular components: cytoplasm, nucleus, nucleolus, mitochondrion, and ER. Denote $L(P)$ to be the set of cellular components protein $P$ belongs to. We construct the following aggregation features:
\[ N_k = |\{P, Q|k \in L(P), k \in L(Q), P \prec Q\}| \]

where \( k \) is a cellular component. \( N_k \) the number of protein pairs in the set where both proteins localizes to \( k \). We also use the fraction of such protein pairs among all pairs, \( N_k/C(n, 2) \) as another feature. We also count the number of protein pairs in the set where the two proteins appear in none of the cellular components together and the fraction of such pairs among all protein pairs as two features.

Finally, we include the number of proteins and number of protein pairs in the set as two global features, which are independent of the signals and data sources. We end up with a total of 134 features, which are richer and are not simply the linear combination of the signals.

**Learning with RankBoost**

RankBoost [40] tries to learn a mapping \( g() \) from a set of proteins to a coherence value, such that \( A \) should be more coherent than \( B \) if \( g(A) > g(B) \). Each training instance \((A \succ B)\) is associated with a loss function \( \text{logit}(g(B) - g(A)) \). Our learning algorithm tries to minimize total loss, which is the sum of the loss functions over all training instances. In our work, \( g() \) is a weighted sum of decision stumps, which are incorporated one at a time to minimize the total loss. In detail, we start with \( g^{(0)}() = 0 \). At each iteration, we pick a decision stump \( d_t() \):

\[
d_t() = \begin{cases} 0 & \text{if } f_i \leq s \\ 1 & \text{if } f_i > s \end{cases} \quad (4.1)
\]

where \( f_i \) is a feature and \( s \) is the threshold of the decision stump. We pick the feature and the threshold to minimize the loss function with the current \( g^{(t)}() = g^{(t-1)}() + d_t() \) over the training set. We repeat the iterations 100 times: \( g() = g^{(100)}() \), and use this final classifier to predict how much a protein set looks like a complex.
Constructing the training set

Since we use RankBoost (see below) to do the learning, each of our training instances involves two sets of proteins with one set ranked higher to the other. For each reference complex $C$ in the training set, we construct the following training instances:

\[
C \succ C \cup P_i, \quad \forall i, s.t. P_i \notin C
\]

\[
C \succ C \setminus P_i, \quad \forall i, s.t. P_i \in C
\]

This is based on the intuition that a reference complex is ranked higher than any protein set that is one-away from the complex. We end up with 746,300 training instances.

Identifying complexes

We use greedy hill climbing over the surface of the coherence function $g()$ until it stops at some local maximum, which we use as our predicted complexes.

In particular, we start with a pair of proteins as the current set. At each step, we consider all the candidate sets that are one-away from the current set — either by adding one outside protein or by removing one existing protein. Among those sets, we pick the one with the highest coherence and use it as the current set. We repeat the steps until the current set has the highest coherence among all the candidate sets, at which point we use the current set as our predicted complex.

4.5.2 Protein-complex model

In the protein-complex model, we try to learn the coherence between a protein $Q$ and a set of proteins $C$, which measures how likely the protein $Q$ is to belong to the same complex as the set $C$. 
Feature construction

We construct the same set of features as in the complexness model (Section 4.5.1) except that we consider pairs of proteins between $Q$ and every protein in $C$. In contrast, the complexness model considers all pairs of proteins within a set. Also, instead of having two global features, we use number of proteins in $C$ as our only global feature that does not depend on any data source.

Learning with LogitBoost

Boosting [41, 24] is a class of algorithms that iteratively combines weak learners to give an ensemble for our classification task. Each weak learner is a simple classifier, such as a decision stump (Eq. (4.1)), that may only weakly correlate with the labels. After a weak learner is trained, we add it to the ensemble with appropriate weight. In the next iteration, the algorithm puts more weights on the data points that are classified incorrectly by the current ensemble, which the next weak learner will focus on. Boosting is able to perform automatic feature selection and has better or comparable accuracy with other state-of-the-art classifiers such as support vector machines (SVMs) [128] in many domains. We implemented a version of boosting algorithms called LogitBoost [24] that uses decision stumps as weak learners and the logit function as the loss function. This variant is shown to be more robust to outliers and overfitting than the standard AdaBoost variant [41]. Our experiments (data not shown) showed that this method performs well on our data, compared to other versions of Boosting and other classification algorithms such as logistic regression and SVMs.

We applied the LogitBoost to the training set and learned a classifier that computes an affinity value between a protein and a set of proteins. The higher the affinity, the more likely the protein should belong to the same complex as the set of proteins.

Constructing the training set

For each reference complex $C$ in the training set, we construct the following training instances:
Positive instances. \((P, C \setminus P) | P \in C\)

negative instances. \((P, C) | P \notin C\)

This is based on the intuition that the pair that consists of a protein \(P\) in the complex \(C\) and the set of remaining proteins in the complex should be in the positive set because both the protein and the set belong to the same complex. On the other hand, a protein \(P\) outside a complex \(C\) and the complex should be in the negative set because the protein and the the set (complex \(C\)) do not belong to the same complex. We end up with 746,300 positive and negative training instances.

Identifying complexes

We start with a pair of proteins as the current set. At each step, we consider each protein outside the current set and use the learned boosting classifier to compute the affinity between the protein and the current set. We add the protein to the current set if the affinity is positive. We also compute the affinity between each protein in the current set with the rest of the proteins and remove the protein from the current set if the affinity is negative. We repeat the steps until no protein can be added or removed, or when a cycle is detected such that we end up with a current set that is identical to the set in some previous iteration.

4.5.3 Protein-protein model

Constructing co-complex protein pairs

The set of positive co-complexed protein pairs consists of all protein pairs that appear in the same complex in the reference set. For the negative set, we first consider all protein pairs \((P, Q)\) such that \(P\) is in a reference complex and \(Q\) outside any version of that complex, in any of the three hand-curated set; we then exclude any pair that is within some other reference complex. The result of this process is 5065 positive pairs and about 1 million negative pairs. Since there are too many negative pairs, for computational expediency, we randomly sample one tenth of the negative pairs to be
used in training while setting each negative pair to have ten times the weight of the positive pairs.

**Integrating multiple features using LogitBoost**

We applied LogitBoost to our training set of positives and negatives. We used the 12 signals from various data sources as our 12 features for the cross-validation runs where we train on four-fifth of the reference complexes and test on the remaining ones. We used the signal from small-scale interactions as our additional feature for the run trained on the entire set of reference complexes. The prediction of the learned ensemble classifier on a given protein pair is taken to be the pair’s affinity in the clustering algorithm below. A high-affinity pair is predicted to be more likely to appear in the same complex.

**HAC with Overlap (HACO)**

The HAC algorithm does not allow overlap between predicted complexes. To address the limitation, we extend it to allow overlap. We started with standard HAC algorithm using average linkage [120], which maintains a pool of merging candidate sets where the distance between two non-overlapping sets is:

\[
d(A, B) = \frac{1}{|A||B|} \sum_{P \in A, Q \in B} d(P, Q)
\]

In our setting, we take \(d(P, Q)\) as the negative of the affinity between protein \(P\) and protein \(Q\). Note that \(d(A, B)\) is the average of the edge distance between proteins in \(A\) and proteins in \(B\).

In HAC, at each step, we pick the two non-overlapping sets with the closest distance, \(A\) and \(B\), and merge them to create a new set, \(M\). \(M\) is added to the pool, while the sets \(A\) and \(B\) are removed. Therefore, in later steps, we could only consider the superset \(M\), and would never be able to use \(A\) or \(B\) again to merge with some other set. Assume that there is another set \(C\) whose distance to \(A\) is only slightly larger than \(d(A, B)\). In this case, the decision to merge \(A\) with \(B\) rather than with \(C\) is arbitrary and unstable. When the actual clusters overlap, a more appropriate solution
Figure 4.1: Illustration of the HACO intuition. Set $A$ is merged to form set $M$ in an earlier step of HACO. Instead of removing $A$ from the candidate pool, as in standard HAC, we keep it and consider its proposed merger with another set $C$. If $N = A \cup C$ is almost as coherent as $M$, as shown on the left panel, we merge $A$ and $C$ to create $N$ so we have overlapping sets of $M$ and $N$. On the other hand, if $N$ is much less coherent than $M$, as shown on the right panel, we do not merge $A$ and $C$. Instead, we consider the potential merge between $M$ and $C$.

would be to have two overlapping merged candidates: $M = A \cup B$ and $N = A \cup C$. We adapted HAC to accommodate this intuition. We define the divergence between $A$ and $M$ as a measure of the cohesiveness of the set $M$ outside of $A$ (Fig. 4.1):

\[
\text{divergence}(A, M) = \frac{1}{|E|} \sum_{(P,Q)\in E, P \prec Q} d(P, Q)
\]

where $E$ is the set of pairs in $M$, but not in $A$: $E = \{(P,Q)|(P,Q) \in (M \times M) \setminus (A \times A), P \prec Q\}$.

If $M$ is not overlapping with $C$, we have the choice of whether to use $A$ or $M$ to merge with $C$. If $\text{divergence}(A, N) - \text{divergence}(A, M)$ is small, it makes sense to merge $A$ and $C$ to create a new set $N$ that is almost as coherent as $M$. On the other hand if the difference is large, we would prefer to replace $A$ with its superset $M$ as the merging candidate to $C$. 

N is almost as coherent as M
In practice, we use $d(A, C)$ to approximate $\text{divergence}(A, N)$: we check whether $\Delta = d(A, C) - \text{divergence}(A, N)$ is small. $\text{divergence}(A, N)$ is the weighted average of $d(A, C)$ and $d(C)$, the distance within $C$. $d(C)$ tends to be smaller than $d(A, C)$ because pairs within $C$, which is formed earlier by some merging, is more coherent than pairs between $A$ and $C$. Therefore, $d(A, C)$ tends to be smaller than $\text{divergence}(A, N)$ so keeping $\Delta$ small is a more stringent requirement to make sure $N$ is almost as coherent as $M$. Moreover, by forcing $d(A, C)$ to be small, we make sure the set $N$ is coherent not just because $d(C)$ is small. With this consideration, we defined the modified distance between $A$ and $C$ to be (Fig. 4.1):

\[
d'(A, C) = \begin{cases} 
    d(A, C) & \text{if } \Delta < \rho \\
    \infty & \text{if } \Delta \geq \rho
\end{cases}
\]

The modified distance $d'$ is used to pick the two closest sets to merge in the next iteration. If $\Delta$ is smaller than a margin, we make $d'$ equal to $d$ and thus allow $A$ and $C$ to merge. On the other hand, if $\Delta$ is large, we make $d' = \infty$ and thus prohibit $A$ and $C$ from merging, in favor of merging their supersets. $\rho$ is the margin parameter: the larger the margin $\rho$, the more likely a set $A$ is to be re-used, resulting in more overlapping subsets constructed by the algorithm. If the margin is 0, it reduces to the standard HAC. Therefore, our HACO algorithm is a generalization of the HAC. Note that we can eliminate a set from the merging candidate pool when its modified distances to all other sets are $\infty$. Of course we can define other modified distance as long as it is larger when $\Delta$ is large and close to $d(A, C)$ when $\Delta$ is small.

In practice, $A$ might have multiple supersets in the pool. Therefore, we look at all of $A$'s supersets in the pool that are not overlapping with $C$ and use the set $M^*$ with smallest divergence from $A$, i.e., the one that provides the best replacement for $A$ in terms of the proposed merger with $C$:

\[
M^*_{A,C} = \arg\min_{M^* \text{ s.t. } A \subseteq M, C \cap M = \emptyset} \text{divergence}(A, M)
\]

We do the same thing with $C$ for its proposed merger with $A$:
M^*_C,A = \arg\min_{M' \text{s.t.} \ C \subset M', A \cap M' = \emptyset} \text{divergence}(C, M')

The smaller of \text{divergence}(A, M^*_{A,C}) and \text{divergence}(C, M^*_C,A) is used to compute the modified distance.

The algorithm terminates when there are no more non-overlapping sets to merge. The output is a cluster-lattice, where the same cluster can be a child of multiple parents in the lattice. The lattice is cut at a certain threshold to generate a set of overlapping clusters. These predicted clusters are the sets that are still in the candidate pool when the distance in the merging process reaches the threshold.

Learning and inference

We applied the HACO algorithm on the pairwise affinity network learned using LogitBoost. We use the same training set in all steps of our pipeline and evaluate the final predictions on complexes in a separate test set which is hidden during all steps of the training process.

We select the threshold that cuts the cluster-lattice in HACO by maximizing the coverage (see Section 4.6.2) on the training set. To pick the margin \( \rho \) in HACO, we cannot use coverage alone because our model would always prefer a bigger margin that keeps more sets in the pool. Therefore, we choose \( \rho \) by maximizing the product of coverage and sensitivity (see Section 4.6.2) on the training set. This approach trades off between the match with the reference set and the number of predicted complexes.

4.6 Experiment setup

4.6.1 Training and test regime

To evaluate our prediction accuracy against the reference set, we divide the 340 reference complexes into five disjoint subsets, or folds. For each fold in the five-fold cross validation, we hide one set and use the remaining four sets to train the model. We then evaluate our predictions on the held-out set, which is not seen during the
Figure 4.2: **Metrics for overlap between two complexes.** $R_j$ is a reference complex and $C_i$ is our predicted complex. The overlap between the two complexes can be quantified by the Jaccard coefficient and the Hamming distance as illustrated in the figure. The better the overlap, the greater the Jaccard coefficient and the smaller the Hamming distance, with perfect match has a Jaccard coefficient 1 and Hamming distance 0.

**Jaccard coefficient:**

$$|C_i \cap R_j| \div |C_i \cup R_j| = \frac{3}{5}$$

**Hamming distance:**

$$|C_i \cup R_j| - |C_i \cap R_j| = 5 - 3 = 2$$

training process. We use the same training-test divide in all steps of our three models: complexity model, protein-complex model, and protein-protein model.

To evaluate our predictions against external data sources, such as biological coherence and essentiality, we augment our model with a signal constructed from small-scale physical interactions and train it on the entire set of 340 reference complexes. To avoid circularity between signals and evaluation, we do not evaluate the predictions from such runs against the reference complexes.

### 4.6.2 Evaluation metrics

To evaluate the matching between reference complexes and predictions, we quantify the overlap between a reference complex $R$ and a predicted complex $C$ in several ways [78] (Fig. 4.2):

**Jaccard coefficient ($JC$):**

$$|R \cap C| / |R \cup C|$$
Hamming distance: $|R \cup C| - |R \cap C|$

Two complexes that overlap well will have a large Jaccard coefficient and a small Hamming distance, with a perfect match has a Jaccard coefficient 1 and Hamming distance 0.

We define the coverage and sensitivity of a set of predictions so we can systematically evaluate genome-wide predictions. For each reference complex, we find the prediction that has the highest Jaccard coefficient. We define the scaled Jaccard coefficient: $SJC(R, C) = \max\{0, 2JC(R, C) - 1\}$. We truncate the value at 0 because it may represent random overlap. We define the coverage as the average Jaccard coefficient per reference complex:

$$\frac{1}{m} \sum_{i=1}^{m} \max_{j=1}^{n} SJC(R_i, C_j)$$

where $m$ is the number of reference complexes and $n$ is the number of predicted complexes.

For sensitivity, we sum the Jaccard coefficients of all the overlapping (reference, prediction) complex pairs, and normalize by the total number of predicted complexes:

$$\frac{1}{m} \sum_{i=1}^{m} \sum_{j=1}^{n} SJC(R_i, C_j)$$

4.7 Results

4.7.1 Coverage and sensitivity of predicted complexes

We compiled a reference set of complexes from MIPS [97], SGD [23], and hand-curation, which is more comprehensive than previous studies [57, 109]. Although it still contains noise and bias, it provides us with the ultimate evaluation of our predictions. There are 340 complexes in our reference set with an average of 4.9 proteins per complex.

We tested our approach using a standard 5-fold cross-validation regime, training on 80% of the complexes and testing on the remaining 20%; the test set was not
Figure 4.3: Size distribution of reference and predicted complexes. Shown here are number of complexes (y-axis) within different size bins (x-axis). Blue bars are for reference complexes and red bars are for predicted complexes.

used in any aspect of the training of the model. In the protein-protein model, which achieves the highest accuracy, we apply HACO to the affinity measure learned using the boosting model on the training data. We evaluate the resulting clusters on the hidden test set. We predicted 417.8 complexes per fold with at least two proteins for each complex. Each complex contains 4.30 proteins on average (Fig. 4.3).

We define a complex to be well-predicted if it is within Hamming distance of 2 to some predicted complex. However two small complexes can be quite different even if their Hamming distance is 2. Therefore we also require the Jaccard coefficient, which takes into account the size of the complexes, to be above 0.5. We also measure the coverage and sensitivity of the set of predictions (see Section 4.6.2): coverage measures
how well the reference set is covered by our predictions and sensitivity measures how well each predicted complex overlaps with the reference set, a measure that takes into consideration the number of predicted complexes.

We compared the different algorithms we used to construct complexes, including the MRF model in the previous chapter. As we can see from Fig. 4.4, the MRF model has a low coverage because it does not predict enough complexes. The complexness model fails to work and its predictions match few reference complexes. Therefore, we do not show the results here. The protein-complex model works reasonably well. It predicts 86 reference complexes perfectly and 45 1-aways (Fig. 4.4). The protein-protein model with HAC is more accurate; it predicts 88 reference complexes and 56 1-aways. See Discussion (Section 4.8) for why the three other models does not work as well as the protein-protein model.

The protein-protein model works the best so we use its results to do the subsequent analysis. It is also the fastest among all our methods because other methods require a separate run to identify each complex.

We compared our results to those predicted by Bader and Hogue [10], Gavin et al. [44], Krogan et al. [79], Hart et al. [57], and Pu et al. [109]. Each method made different decisions for defining the affinity function and for clustering it.

Fig. 4.5 shows the number of reference complexes that is well matched by some predicted complexes and total number of complexes we predicted. Fig. 4.6 shows the coverage, sensitivity, and their product. As we can see, our protein-protein model achieves significantly better results than any of these methods; the results are better even when we use simple HAC for the clustering, and improve further when we use HACO. We note that Hart et al. and Pu et al. are the state-of-the-art in complex predictions and have been extensively compared with other complex prediction methods. HACO was able to perfectly recover 42% and 46% more reference complexes compared with the Hart et al. and Pu et al. respectively. The corresponding increase in sensitivity is 6% and 29% respectively and increase in coverage is 28% and 33% respectively. The results suggest that these improvements are a consequence of our use of data integration with state-of-the-art machine learning. In particular, the Pu method and the Hart method, both of which used MCL applied to different
Figure 4.4: **Prediction accuracy of our different models.** We compare our predicted complexes with the reference complexes. x-axis are the different models we tried. MRF is the model we constructed in the last chapter. Protein-complex model tries to learn an affinity function on how likely a protein should belong to the same complex with a set of proteins. Protein-protein model is our two-stage approach where we first learn an affinity function on how likely two proteins belong to the same complex. Then, we cluster the pairwise affinity network using HAC and treat the resulting clusters as our predicted complexes. The y-axis is the number of reference complexes that are well matched by our predictions. Blue bars are for reference complexes that are perfectly matched by our predictions. Green bars are for reference complexes that differ with some of our predicted complexes by one protein, either one extra or one fewer. Red bars are for reference complexes that differ with some of our predicted complexes by two proteins. As we can see, the protein-protein model performs the best. The complexness model has low accuracy so the result is not shown here.
Figure 4.5: **Accuracy in reconstructing reference complexes.** We compare our predicted complexes to other state-of-the-art methods in the ability to accurately reconstruct reference complexes. x-axis is the different methods we compared. The y-axis is the number of reference complexes that are well matched by our predictions. Blue bars are for reference complexes that are perfectly matched by our predictions. Green bars are for reference complexes that differ with some of our predicted complexes by one protein, either one extra or one fewer. Red bars are for reference complexes that differ with some of our predicted complexes by two proteins. As we can see, Hart et al. and Pu et al. are state-of-the-art methods that outperform Gavin et al. and Krogan et al.. Bader et al. have even lower accuracy, which is not shown here. Applying HAC to PE score (HAC PE) performed slightly better than Hart et al. and Pu et al., which use MCL. Our protein-protein model is able to achieve significantly better results than any other method by integrating multiple sources of data. The results are better even when we use simple HAC (HAC all, 88 perfect matches) for the clustering, and improve further when we use HACO (HACO, 95 perfect matches). In ‘Train all’, we trained on all data and tested on the same data. Its accuracy is only slightly better, which indicates little overfitting.
Figure 4.6: **Coverage and sensitivity of predicted complexes.** x-axis is the sensitivity of our predictions, which quantifies how likely a prediction matches some reference complexes. y-axis is the coverage of our predictions, which quantifies how many reference complexes are matched by our predictions. See text for the exact definition. All the results are based on cross-validation except for ‘Train all’, which is trained and tested on all data using Boosting and HACO. Our protein-protein model (HAC all or HACO) has higher sensitivities and coverages than other methods. HACO has the highest product of sensitivity and coverage, except for ‘Train all’.
affinities obtained from the TAP-MS data, performed very similarly. Interestingly, HAC applied to the PE score performed slightly better than MCL applied to the PE score (HAC-PE vs. Pu et al.). These three methods performed better than Bader et al., Gavin et al., and Krogan et al., which can be explained by the fact that these earlier methods used only a single set of purifications. These results demonstrate the importance of combining data from multiple data sources, integrated appropriately.

The HACO algorithm helps address several of the limitations of the HAC approach. First, it reduces the sensitivity of the complex definitions to a single universal threshold in the hierarchy. One such example involves the 15-protein SAGA complex. Here, HAC predicts a 24-protein superset of the SAGA complex. This cluster is a much weaker cluster than SAGA itself: the average affinity between the SAGA proteins is 0.35, as compared to the average affinity, -1.19, for pairs within the 23 proteins excluding pairs of SAGA proteins. By comparison, HACO, by keeping multiple hypotheses relative to the cutoff, predicted both a 23 protein cluster (similar to the HAC prediction), but also predicted the subcluster that corresponds perfectly to the SAGA complex. The second limitation addressed by HACO is that it avoids an early commitment to incorrect outcomes. For example, the affinity between RAD23 and PNG1 is slightly higher than that between RAD23 and RAD4. HAC incorrectly merges RAD23 and PNG1, and now cannot reuse RAD23 in any other complex. HACO can reuse RAD23, merging it with RAD4 to create a complex that perfectly matches the NEF2 (nucleotide-excision repair factor 2) complex in the reference set.

4.7.2 Contribution of each data source

Given the importance of data integration, it is useful to see which data sources play the most important role in our results. We first considered the contribution of each feature to our learned affinity function. Our approach uses LogitBoost [24], which defines the affinity function as the weighted sum of many weak learners, each of which is a decision stump on one of the features. The top weak learners involve features that are deemed to be most predictive. The top weak learners in the order of their importance use: correlation of PE score (weight 3.84); semantic similarity in the
truncated GO cellular component categories (-2.2); directed PE score (0.58); small-scale physical interactions (0.55); and co-expression (0.16). It is interesting to note that the correlation of the PE score is deemed more informative than the PE score itself. One explanation is that the pairwise PE score between proteins $P$ and $Q$ is still a noisy measure for co-complexness, but if two proteins are truly co-complexed, they are likely to have similar interactions with other proteins.

To assess the contribution of each data source, we successively applied our pipeline with HAC to the data source alone and to all data sources except that data source. As we can see from Fig. 4.7, the PE score plays the dominant role and by itself predicts most of the complexes. Importantly, our method here combines different variations of PE score (direct, indirect, scaled, total, and correlation) using boosting, generating an affinity score that is quite a bit better at predicting complexes than the original scaled PE score (73/54/16 perfect matches/1-away/2-aways for ‘HAC PE’ in Fig. 4.5 versus 81/50/19 for the PE-based features alone in Fig. 4.7). This result demonstrates the value of applying machine learning methods specifically optimized for the problem of complex identification. Nevertheless, we still get a significant improvement by integrating in other data sources.

Localization and expression have a similar effect. By itself, neither predicts any complexes at all; this is not surprising, as both are features with low precision. However, removing each of them decreases the accuracy, suggesting that they provide a signal that is independent of the PE score, and can help resolve some of its ambiguities and errors. The yeast two-hybrid feature has the opposite behavior: In isolation, it predicts a reasonable number of complexes; however, removing it does not decrease accuracy at all. This behavior can be explained by the hypothesis that yeast two-hybrid data largely correlates with PE score; thus, although it is predictive, it does not add much given the PE score data. This last hypothesis is further verified by the fact that localization and expression features appear within the top 5 weak learners whereas the yeast two-hybrid feature does not.
To assess the contribution of each data source, we successively applied our pipeline with HAC to the data source alone and to all data sources except that data source. The x-axis shows the runs with interesting observations. The y-axis is the number of reference complexes that are well matched by our predictions. Blue bars are for reference complexes that are perfectly matched by our predictions. Green bars are for reference complexes that differ with some of our predicted complexes by one protein, either one extra or one fewer. Red bars are for reference complexes that differ with some of our predicted complexes by two proteins. As we can see, the PE score by itself predicts most of the complexes. Nevertheless, we still get a significant improvement by integrating other data sources. Localization or expression by itself does not predict any complexes at all, but removing it decreases the accuracy. On the other hand, the yeast two-hybrid by itself predicts a reasonable number of complexes, but removing it does not decrease accuracy at all.
4.7.3 Biological coherence of predicted complexes

We also evaluated the validity of our predicted complexes by comparing to external data sources not used in the training and not directly related to reference complexes. Here, we train on all reference complexes with the additional feature from the small-scale experiments and predicted 383 complexes. For all biological coherence validations, we compute the coherence for each complex as the average of the coherence measure for all pairs in the complex. Then, we take the average across all complexes predicted. We compare to the methods of Hart et al. [57], and Pu et al. [109], which consistently out-performed all previous methods. As a different benchmark, we also compare to the coherence for the highest-affinity protein pairs (those that are most likely to belong to the same complex).

We validate our predictions by looking at various measures of biological coherence (Fig. 4.8): similarity of GO biological process; similarity in the level of protein abundance for different complex components; correlation of growth defect profiles across a broad range of conditions; and co-regulation, as measured by sharing of transcription factors. For all measures, HACO with our affinity function considerably outperformed all other approaches, with the method of Hart et al. being the closest competitor.

- GO data were downloaded on 25 June 2007. We compute the semantic distance between two proteins as the log size of their smallest common category [89] in the Biological Process hierarchy.
  
  Our complexes have a semantic distance 8% and 17% lower than the methods of Hart et al. and Pu et al. respectively. Conversely, they are 21% less coherent than the reference complexes.

- We downloaded the protein abundance data from [47]. We use log of measured protein levels in terms of molecules per cell as the protein abundance value.

  The improvement in coherence of protein abundance is 5%, 10%; notably, here our predicted complexes outperform the reference complexes by 2% on this metric. However, our complexes are 12% less coherent than the top affinity pairs, suggesting that proteins with lower affinity scores can be members of the
Figure 4.8: **Coherence of our predicted complexes.** We computed the functional coherence between proteins in the same complex against external data sources that are not used in training. More coherent proteins have a smaller difference in protein abundance and a smaller distance in GO biological process. On the other hand, more coherent proteins share more transcriptional regulators and have a higher growth fitness correlation. The y-axis shows the values for above measurements of functional coherence. The highest point of the y-axis is the coherence for random protein pairs in the case of protein abundance and GO biological process. The lowest point of the y-axis is the coherence for random pairs in the case of regulator overlap and growth correlation. As we can see, our predicted set of complexes is able to outperform other state-of-the-art methods. It is even better than the reference set of complexes in two cases and only marginally worse in another case. For the remaining case regarding GO biological process, we are worse only because the reference complexes and GO annotation are derived (at least partly) from similar data sources.
complex, but also play other roles in the cell, reducing their correlation with other proteins in the same complex.

- The growth phenotype data was obtained from [58]. For each gene, its homozygous deletion strain is grown in 418 experiments with different drug treatments. The log ratio of the deletion strain’s growth in no-drug control to its growth with the drug treatment is used to define the growth phenotype in that particular condition. For each pair of genes, we compute the Pearson correlation of the growth phenotypes across all 418 conditions. Based on this measure, our predicted complexes are 19%, 31% more coherent than Hart et al. and Pu et al. respectively, a very significant improvement. They are 4% worse than the reference complexes.

- We downloaded the transcriptional regulation data from [91, 56]. We used p-value cutoff at 0.001 and required conservation across species to define the transcription factors for each protein. Protein pairs within our complexes on average share 30%, 59% more transcription factors then those in Hart et al. and Pu et al. respectively. They share 17% more transcription factors than those in the reference complexes.

The comparison to the reference complexes is also interesting: Our complexes are 17% more coherent than the reference complexes on regulator overlap, and perform similarly on correlation of abundance and growth phenotype (within ±4%). Conversely, our complexes are 21% less coherent than the reference complexes on GO biological process annotations; this is not surprising, as the reference complexes and GO annotations are derived (at least partly) from similar data sources, such as literature and small scale experiments. Overall, when comparing to data sources that were not used in constructing the reference complexes, our predictions seem to perform as well or better than the reference set, suggesting that our predictions provide a strong set of complexes that can be used as a new reference.
4.7.4 Essentiality and complex size

Much discussion has occurred regarding the relationship between essentiality and the structure of the protein-protein interaction network. Early work of Jeong et al. and Han et al. [66, 55] found that hub proteins in a protein-protein interaction network are more likely to be encoded by essential genes. However, a much deeper insight on the relationship between the protein network and essentiality can be obtained by considering the network at the level of complexes rather than pairwise interactions. Such an analysis was recently performed by Hart et al. [57], who showed that essential proteins are concentrated in certain complexes, a phenomenon also found in our predicted complexes (Fig. 4.9). Thus, we have a dichotomy of essential and non-essential complexes. However, that finding does not explain why ‘hubs’ in the network are more likely to be essential. We therefore looked into the distribution of essential proteins in complexes of different sizes. As we can see from Fig. 4.10, the larger the complex, the greater the proportion of essential proteins among its components. This finding suggests that ‘hubs’ in the protein interaction network are generally components in a large complex, and the finding regarding the essentiality of hubs arises from the fact that large complexes are more likely to have a much higher ratio of essential genes.

Indeed, we found that essentiality is better explained by complex size than hubness. We rank every protein based on the size of the largest complex to which it belongs, and for the $K$ top-ranked proteins (for different values of $K$), plot the number of essential vs. non-essential proteins (Fig. 4.11). We plotted a similar curve by using the hubness of the protein — the degree in the yeast two-hybrid protein-protein interaction network [63, 127]. As we can see, complex size is a much better predictor for essentiality than hubness. We note that if we use the scaled PE score (at threshold $\phi > 0.5$) to define a protein-interaction network, the hubness becomes a strong predictor of protein essentiality. However, PE score is more related to co-complexness than interaction, and thus this metric of hubness is directly related to complex size. Nevertheless, using complex size directly is still better than using scaled PE score. Interestingly, if we use the size of the largest enclosing reference complex to rank each protein, the result is slightly less predictive than using our predicted complexes, or
CHAPTER 4. STOICHIOMETRICALLY STABLE COMPLEXES

Figure 4.9: Proportion of essential proteins across complexes. For each complex, we compute the proportion of its protein members that are essential. The x-axis is a specific bin and the y-axis is the number of complexes whose proportion of essential proteins falls into that bin. As we can see, most complexes have either few essential proteins (<10%) or almost all essential proteins (>90%).
Figure 4.10: **Relationship between complex size and essentiality.** We look at the relationship between size of the complexes and the proportion of essential proteins in the complexes. The x-axis is the size bin of the complexes. The y-axis is the proportion of essential proteins in all complexes within the size bin. As we can see, larger complexes tend to have a higher proportion of essential proteins.
Figure 4.11: Explaining essentiality using complex size vs. hubness. We try to explain the essentiality of a protein by using the size of the largest enclosing complex vs. by using its hubness in the protein-protein interaction network. We rank the proteins based either on complex size or on hubness. The x-axis is the number of essential proteins in the $K$ top-ranked proteins (for different values of $K$) and the y-axis is the number of non-essential proteins. For the red curve, we use the size of the predicted complexes while the light blue curve uses the size of the reference complexes. For the blue curve, we use the hubness — degree in the yeast two-hybrid protein-protein interaction network. The green curve also uses the hubness but in a network whose edges have scaled PE score $>0.5$. As we can see, complex size is a much better predictor for essentiality than hubness (blue curve). Since the PE score is more related to co-complexness than interaction, the hubness based on PE score (green curve) is directly related to complex size. Nevertheless, using the size of predicted complexes directly (red curve) is still better than using the PE score. Interestingly, using the size of reference complexes (light blue curve) is slightly less predictive than using the size of our predicted complexes, or even the PE score directly.
even the scaled PE score directly.

4.8 Discussion

Identifying a comprehensive set of protein complexes in yeast is an important but challenging task. The high-quality and high-throughput TAP-MS data, which directly measures co-complexness, provides a starting point for accurately reconstructing these complexes. Indeed, two recent studies [57, 109] used the TAP-MS data to produce a set of complexes with the state-of-the-art performances. Both methods applied a simple clustering algorithm to a score derived directly from the TAP-MS data.

In this chapter, we tried four different methods and the protein-protein model is able to significantly improve the accuracy of the complex reconstruction in three ways. First, we carefully constructed a large set of reference complexes and trained our model so it specifically predicts co-membership in stoichiometrically stable complexes. Second, we integrated multiple sources of heterogeneous data so our predictions are more robust to noise and incomplete coverage in the TAP-MS data. Finally, we extended the highly effective hierarchical agglomerative clustering (HAC) algorithm to allow reconstruction of clusters with overlap, a flexibility that allows it to circumvent many of the limitations of the standard HAC algorithm. We show that the resulting set of predicted complexes (available from our website [4]) has significantly higher accuracy and is more biologically coherent than that of other recent methods. In many cases, it is even more coherent than the reference set, indicating it is of high quality and can be used as a new reference set. When combined with our comprehensive, hand-curated reference set (also available from our website), our work provides a significant new resource to the research community.

The other three methods we tried to reconstruct complexes all seem to have reasonable intuition behind them. It turns out, however, both the Markov Network model and complexity model has low accuracy. We looked more carefully into the reasons why they do not work. After a certain number of complexes predicted, the Markov Network model mostly constructs complexes that are identical to the ones discovered before. Therefore, we end up with a limited number of complexes
predicted. As for the complexness model, it tries to compute a coherence value for a set of proteins based on how coherent its features are such as PE score and co-expression. However, if a complex is coherent, its subsets, especially those closely connected components, are also likely to be coherent. Therefore, our RankBoost model is unable to distinguish a complex from its subsets, which severely limits its ability in identifying complexes.

Both the protein-complex model and protein-protein model achieved good accuracy, with the latter being unequivocally better. One problem for the protein-complex model is that at each iteration, it considers every protein outside the current complex to be a possible candidate to be added. Since there are thousands of proteins outside the complex, the chances that at least one of them would have a positive affinity with the current set is high, even if we have low false positive rate. Therefore, even if the current set is already a complex, we are still likely to add in extra proteins, which causes the prediction to be not perfect. On the other hand, in the protein-protein model, at each step of the HAC, we consider merging two sets instead of individual proteins. So if some set in the current pool is already a complex, it will be considered to merge with other sets in the pool only, which is a relatively small number, and since we use average linkage, one false high affinity between the two sets would be averaged out by other pairs with low affinities. Therefore, our set is less likely to be merged with other sets that are not in the same complex, and thus the prediction would be perfect. As for HACO, we prefer to use the superset as the merging candidate unless the subset satisfy some stringent criteria. This also limits the choices a set has as its merging candidate.

There are still many reference complexes that are not matched by our predicted complexes. Many of them fall into roughly two categories. In the first category, proteins in the reference complex have high affinities with each other, and are grouped as a set during the HACO procedure. However, they are not selected in our predictions because they are not at the granularity where we cut our HACO cluster-lattice. They then become subsets or supersets of some predicted complexes. In fact, if we use all the sets generated during our HACO procedure as predicted complexes, 136 reference complexes would be perfectly predicted and 243 would be well matched by some
predicted complexes, in comparison to 95 perfect matches and 189 well matched by our current predictions. However, this approach would result in far too many predictions (3478), greatly reducing sensitivity. This fact highlights the limitations in defining a universal level of affinity at which one determines that a group of proteins form a stable complex, and suggests that a more flexible technique may be a useful direction for future work. In the second category, the proteins in the reference complex do not have high affinities with each other. This situation arises when the signal in the data is not sufficiently strong to indicate that two proteins are likely to interact. As most of our signal comes from the TAP-MS data, such ‘blind spots’ can arise from limitations of this assay, such as complexes of low abundance or that are membrane-bound. In particular, we note that the TAP-MS data was all acquired in a single condition (rich media), and some complexes may simply not be present in the cell in that condition. Our inability to recover such complexes arises not from computational limitations, but from limitations in the data. New experimental assays are needed before these complexes can be reconstructed.

Like other previous approaches, our method was developed in the context of S. cerevisiae, where we have the most data relevant to protein-protein interactions. Having a high-quality set of predicted complexes is of significant value even in yeast, as many key complexes are conserved from yeast to human. Moreover, our method is general-purpose, and can easily be applied more broadly. With the increasing amount of high-throughput protein-protein interaction data, both TAP-MS [37] and other assays [108, 123], we should soon be able to provide a high-quality reconstruction of protein complexes in other organisms, including human.
Chapter 5

Complex-complex interactions

In this chapter, we use the complexes we predicted in the previous chapter as basic units, and predict interactions between them. Complexes that interact with each other are more likely to be involved in the same biological pathway. This puts proteins or complexes into a larger context for us to understand how they influence cellular processes. In the end, we create a unified network of interactions between core cellular units — complexes and proteins.

5.1 Introduction

Complexes and individual proteins that act alone are the basic entities, or building blocks, from which the protein interaction network in the cell is comprised. Given the set of complexes we predicted in the previous chapter, which is high-quality and comprehensive, we now try to reconstruct the network of interactions between these entities. Interactions between entities usually happen when they try to coordinate their activities to achieve a certain biological task. For example in a signaling pathway, an entity (a protein or complex) receives signals from an upstream entity, and interacts with a downstream entity to activate or inhibit its function. Once activated or inhibited, the downstream entity passes the signal further down through more interactions. The interactions between upstream and downstream entities usually
involve post-translational modification of the downstream entity, such as phosphorylation or methylation, which triggers a change in its 3D configuration and enables its activities. In general, unlike the interactions between proteins within the same complex, the interactions between complexes are more transient. They happen only in a certain context, involving a specific time, location, or condition. Understanding such interactions is important for understanding cellular interactions and for providing a higher level view of cellular processes.

In this chapter, we reformulate the task of reconstructing the protein interaction network, which is the focus of much prior work. Rather than predicting interactions between individual proteins — a somewhat confusing network that confounds interactions within complexes and interactions between complexes — we tackle the novel task of predicting a comprehensive protein interaction network that involves both individual proteins and larger complexes. We argue that these entities are the right building blocks in reconstructing cellular processes, providing a view of cellular interaction networks that is both easier to interpret than the complex network of interactions between individual proteins, and more faithful to biological reality. Moreover, a complex, which is a stable collection of many proteins that act together, provides a more robust basis for predicting interactions, as we can combine signals for all its constituent proteins, reducing sensitivity to noise.

To accomplish this goal, we construct a reference set of complex-complex interactions, considering two complexes to interact if they are significantly enriched for reliable interactions between their components. We further augmented this set with a hand-curated list of established complex-complex interactions. We then use a machine learning approach to detect the ‘signature’ of such interactions from a large set of assays that are likely to be indicative. We explore different machine learning methods, and show that a partially supervised naive Bayes model, where we learn the model from both labeled and unlabeled interactions, provides the best performance. This model is applied both to our predicted complexes and to individual proteins, providing a new, comprehensive reconstruction of the S. cerevisiae interaction network, which can be downloaded from our webpage [3]. We show that entities that are predicted to interact are more likely to share the same functional categories.
5.2 Related work

Much work has focused on predicting interactions, possibly transient, between proteins. For example, Deng et al. [31] and Liu et al. [88] tried to predict protein-protein interactions by building a graphical model that takes into consideration the protein sequence motifs and observed protein-protein interactions. Bock and Gough [14] used Support Vector Machines (SVM), a supervised learning approach, to predict interactions based on the physicochemical properties of the amino acids on the protein sequence. Importantly, our work predicts interactions between complexes instead of just between proteins. Computationally, this enables us to combine the signals from all the constituent proteins in the complexes, which reduces the sensitivity to noise so the result is more robust. Biologically, this gives a more interpretable interaction network.

Some other approaches mentioned earlier [22, 84, 92, 117, 124, 129, 131, 135, 138] integrate multiple sources of data to predict whether two proteins are functionally related: act in the same complex, pathway, or functional module. By comparison, our work focuses specifically on predicting transiently interacting pairs, which is the first step in identifying co-pathway complexes. It also provide us with a way to understand the internal structure, with which complexes coordinate with each other to execute a pathway.

5.3 Reference list of positive and negative complex-complex interactions

We use the same set of reliable interactions as in Section 2.3.1. We compute the number of reliable interactions between proteins of two complexes, and compared it to what we expect if the reliable interactions are distributed randomly. We define the two complexes to be interacting if the enrichment of reliable interactions is more than 20 standard deviations above the mean. This gives us a list of 82 interactions between the set of 383 complexes we just predicted. To augment this list, we generated a list of 59 additional known interactions between 81 named complexes. Both lists are
available from our website [3]. To avoid the redundancy between those 81 named complexes and our 383 predicted complexes, we replace a predicted complex by a named complex if they overlap with Jaccard coefficient > 0.5. This process gives us a total of 421 complexes with 133 unique interactions between them, which is used as our positive reference set. The named complexes are better known and more thoroughly studied so if a pair of named complexes is not known to interact, they are more likely to be non-interacting. Therefore, we create a negative reference set of 3173 non-interactions by using all pairs of named complexes that are not in our positive set. The interaction status of all the remaining pairs of complexes, named or predicted, is treated as unknown.

We also apply our model to predict a unified interaction network involve both proteins and complexes. In this case, we have both the interaction between two complexes and the interaction between a protein and a complex. We create the positive set using the same procedure as above. As for the negative set, in addition to the above negative reference set between complexes, we randomly sampled 6560 protein-complex pairs that are not in the positive set and added them to our negative reference set. The number 6560 is chosen so the ratio of positive to negative pairs for protein-complex interactions is the same the ratio for complex-complex interactions. All our reference lists are available from our website [3].

5.4 Protein-level signals for predicting complex-complex interactions

Since there is no direct measurement of complex-complex interactions, we try to use as much indirect evidence as possible. Besides all data sources used for identifying complexes, we added four additional data sources based on correlation of growth fitness, correlation of transcription factor profile, protein-protein interaction prediction, and condition specific expression correlation.

The correlation of growth fitness profile [58] is computed as described in Section 4.7.3.
CHAPTER 5. COMPLEX-COMPLEX INTERACTIONS

For each protein, we create a transcription factor (TF) profile vector, where each position in the vector represents a TF and its value is 1 if the TF is found to regulate the protein [91] and 0 if it is not. We used the same transcription regulation data as described in Section 4.7.3. For any pair of proteins, we compute the mutual information between the profile vectors of the two proteins using the method described in Date et al. [29].

There are many works on integrating multiple sources of data to predict protein-protein interactions. In particular, the InSite method [133] in Chapter 2 integrates protein sequence motifs, evidence for protein-protein interactions, and evidence for motif-motif interactions in a principled probabilistic framework to make high-quality predictions of protein-protein interactions. Here, we use the InSite method, but trained without the reliable interactions between complexes in our positive reference set. We use the predicted probabilities that two proteins interact as one more data source.

We processed the expression data in accordance with our intuition that transient interactions occur under specific conditions, and we should only expect expression profiles of interacting proteins to be correlated only when at least one of the pair is active. Specifically, we divided our expression data into 76 conditions [139, 95, 20, 81, 106, 43, 42, 32, 70], each of which represents a particular time course. In accordance with convention, we quantify a protein’s activity under certain condition according to its maximum deviation from norm, or in other words the maximum absolute expression (assuming norm to be 0). For each condition, we define a protein to be differentially expressed, or active, if its maximum absolute expression is above a cutoff, which we specify to be 1.0. For each pair of proteins, we compute Pearson Correlation Coefficient (PCC) separately in each condition. If a protein in the pair is inactive under a condition, the PCC value for the condition is assumed to be 0. We use the PCC value, averaged across all conditions under which at least one protein out of the pair is active, as our last signal. Initial investigation showed that this signal is better correlated with the reference complex-complex interactions than the overall PCC across all conditions. We note that, for the task of predicting when two proteins are co-complexed, the simple correlation performed better (data not shown),
consistent with the fact that the activity of two members of a stable complex is likely to be similar across a wide range of conditions.

\section*{5.5 Aggregating signals into features between complexes}

All forms of signals in our analysis involve a pair of proteins. To predict interactions between two complexes, $C$ and $D$, we aggregate the signals for all protein pairs between $C$ and $D$ and produce the following features:

$$f_{ij} = A_i(\{s_j(P,Q) | P \in C, Q \in D\})$$

where $A_i()$ is some aggregating function, such as: sum, max, mean, min, decayed max, decayed min, etc. We use the same list of aggregating functions as in Appendix A. $s_j()$ represents the $j$'th signal type between a pair of proteins. We also use four global features, independent of the data sources: size of the first complex, size of the second complex, number of protein pairs between the two complexes, and number of overlapping proteins between the two complexes.

The Naive Bayes model that we use assumes all features to be conditionally independent of each other given the status of whether two complexes interact or not. Therefore for each data source, we pick only the best aggregating function in order to reduce the conditional dependencies between the features. To do this, we define $r_{ij}$ to be the area under the ROC curve if we use the feature $f_{ij}$ alone to predict complex-complex interactions. The greater the $r_{ij}$, the stronger correlation between the feature and the complex-complex interactions. Therefore, for Naive Bayes, we use the following features in addition to one of the four global features:

$$f_j = f_{ij} \text{ where } i = \arg\max_i r_{ij}$$

The aggregating functions chosen for our signals and the global feature are listed in Table 5.1.
Table 5.1: **List of aggregating functions chosen.** We plot the ROC curve of each aggregating function, applied to a data source, in predicting interactions and pick the aggregating function that has the maximum area under the curve. The data source is listed in the first column and the corresponding best aggregating function is listed in the second column. Among all global features, we also pick the one that has the maximum area under the ROC curve. The first table is for the complex-complex interaction network and the second table is for the unified interaction network involving both proteins and complexes.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Aggregating Function</th>
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<tbody>
<tr>
<td>Direct PE score</td>
<td>Max</td>
</tr>
<tr>
<td>Indirect PE score</td>
<td>Max</td>
</tr>
<tr>
<td>Scaled PE score</td>
<td>Sum</td>
</tr>
<tr>
<td>PE score correlation</td>
<td>Sum</td>
</tr>
<tr>
<td>GO distance</td>
<td>Min</td>
</tr>
<tr>
<td>Trans-membrane</td>
<td>Sum</td>
</tr>
<tr>
<td>Co-expression (SMD)</td>
<td>Average of top three values</td>
</tr>
<tr>
<td>InSite</td>
<td>Decayed min</td>
</tr>
<tr>
<td>Fitness correlation</td>
<td>Average of top three values</td>
</tr>
<tr>
<td>TF Mutual information</td>
<td>Sum</td>
</tr>
<tr>
<td>Time-series correlation</td>
<td>Max</td>
</tr>
<tr>
<td>GFP localization</td>
<td>Number of pairs with different localization</td>
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<tr>
<td>Global</td>
<td>Number of protein pairs</td>
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<td>Fraction of pairs with different localization</td>
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<tr>
<td>Global</td>
<td>Number of protein pairs</td>
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5.6 Methods

We experimented with different machine learning algorithms for making our predictions: (1) a simple Naive Bayes model, where the effects of different feature types are assumed to be independent; (2) a discriminative boosting algorithm, as we used in predicting co-complexed affinities between protein pairs above; (3) a Naive Bayes model where the unlabeled complex-complex interactions are taken to be unobserved variables, and the model is trained via the Expectation Maximization (EM) algorithm. This last approach is based on the fact that the amount of labeled training data is quite limited in this task, but the unlabeled data also provides us with useful information about the behavior of different features in interacting and non-interacting pairs. A variant of this same approach was used with success in the InSite model [133] in Chapter 2.

More formally, for each pair of complexes, we construct an ‘interaction variable’, whose value is 1 if the two complexes are in the positive reference set of interacting complexes, 0 if they are in the negative reference set, and unobserved otherwise. Each feature of the complex pair is associated with two conditional distributions: one for the case of an interacting and the other for the case of a non-interacting pair. These distributions are defined via some parametric classes (Table 5.2), which are picked by examining the empirical distributions. The distributions for the different features are taken to be independent of each other within each of the two cases. The model is trained via the following EM procedure. We initialize the model parameters to those that would be obtained from MLE estimation using the pairs in our reference set alone. We then iteratively repeat the following two steps until convergence. In the E-step, we use our current model to compute the marginal probability of each unobserved interaction variable given the features associated with the pair. We use the computed probability as a soft assignment to the interaction variable. In the M-step, we learn the parameters for the distributions using the inferred soft assignment to all interaction variables; the variables in the reference set are always fixed to their known value. We use the model obtained at convergence to predict, for each pair of complexes not in our reference set, the probability with which the pair interacts.
### Table 5.2: Parametric family in the model for the complex-complex interaction network

Shown here are the parametric family of distributions for the feature values given the two complexes interact and given the two complexes do not interact. The parametric families are picked based on examining the reference set of complex-complex interactions and non-interactions. The parameters for the distributions are learned from the data. Exponential distribution starts at 0 and goes to the right. 'Reverse exponential' starts at 1 and goes to the left. 'point 0' and 'point 1' refers to discrete distributions with all mass at the point 0 and point 1 respectively.

<table>
<thead>
<tr>
<th>Signal</th>
<th>Distribution given non-interaction</th>
<th>Distribution given interaction</th>
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</thead>
<tbody>
<tr>
<td>direct PE score</td>
<td>Mixture of uniform if negative and exponential if positive</td>
<td>Mixture of uniform if negative and Gaussian if positive</td>
</tr>
<tr>
<td>indirect PE score</td>
<td>Exponential</td>
<td>Exponential</td>
</tr>
<tr>
<td>scaled PE score</td>
<td>Exponential</td>
<td>Mixture of uniform and exponential</td>
</tr>
<tr>
<td>PE score correlation</td>
<td>Mixture of two exponentials with different means</td>
<td>Mixture of two exponentials with different means</td>
</tr>
<tr>
<td>GO distance</td>
<td>Mixture of three uniforms with boundary 6.7 and 7.5</td>
<td>Mixture of three uniforms with boundary 6.7 and 7.5</td>
</tr>
<tr>
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</tr>
<tr>
<td>direct PE score</td>
<td>Mixture of two exponentials with different means</td>
<td>Mixture of exponential and Gaussian</td>
</tr>
<tr>
<td>indirect PE score</td>
<td>Mixture of point 0 and Gaussian</td>
<td>Mixture of point 0 and Gaussian</td>
</tr>
<tr>
<td>scaled PE score</td>
<td>Mixture of point 0 and exponential</td>
<td>Mixture of point 0 and exponential</td>
</tr>
<tr>
<td>PE score correlation</td>
<td>Mixture of two exponentials with different means</td>
<td>Mixture of two exponentials with different means</td>
</tr>
<tr>
<td>GO distance</td>
<td>Mixture of three uniforms with boundary 6.7 and 8.4</td>
<td>Mixture of three uniforms with boundary 6.7 and 8.4</td>
</tr>
<tr>
<td>Trans-membrane</td>
<td>Exponential</td>
<td>Exponential</td>
</tr>
<tr>
<td>Co-expression (SMD)</td>
<td>Mixture of point 0 and Gaussian</td>
<td>Gaussian</td>
</tr>
<tr>
<td>InSite</td>
<td>Reverse exponential</td>
<td>Reverse exponential</td>
</tr>
<tr>
<td>Fitness correlation</td>
<td>Mixture of point 0 and Gaussian</td>
<td>Mixture of point 0 and Gaussian</td>
</tr>
<tr>
<td>TF Mutual information</td>
<td>Mixture of point 0 and exponential</td>
<td>Mixture of point 0 and exponential</td>
</tr>
<tr>
<td>Time-series correlation</td>
<td>Mixture of point 0 and Gaussian</td>
<td>Gaussian</td>
</tr>
<tr>
<td>GFP localization</td>
<td>Mixture of point 0, point 1, and Gaussian</td>
<td>Mixture of point 0, point 1, and Gaussian</td>
</tr>
<tr>
<td>Number of protein pairs</td>
<td>Exponential</td>
<td>Exponential</td>
</tr>
</tbody>
</table>

**Table 5.3:** Parametric family in the model for the unified interaction network. Same as Table 5.2 except here the parametric families are picked based on examining the reference set for the unified interaction network involving both proteins and complexes.
When training using the LogitBoost model, we are not making independence assumptions between the different features. Hence, there we include all features $f_{ij}$, instead of just picking the best aggregating function for each feature type.

We used the same naive Bayes + EM procedure for the protein-complex interaction predictions, although the best aggregating functions picked and the set of parametric classes used for the feature distributions was a little different. (See Table 5.1 and Table 5.3.)

5.7 Results

5.7.1 Accuracy of complex-complex interaction predictions

We compiled a reference set of complex-complex interactions from reliable protein-protein interactions and hand-curation. There are 133 interactions in the positive reference set and 3173 non-interactions in the negative reference set.

We used ten-fold cross-validation to evaluate the ability of our model in accurately predicting complex-complex interactions. We randomly divide our reference interactions into ten sets. In each fold, we hide one set and train on the remaining nine sets. We then make predictions on the held-out set using the learned model. We compare three methods (see Methods): simple Naive Bayes, a discriminative Boosting method, and Naive Bayes with EM (NB+EM) that also makes use of the data for pairs that are not in our reference set. As we can see from Fig. 5.1, ‘NB+EM’ performs better than both other methods, achieving very high performance: 44 of the top 50 predictions (88%) are in the positive reference set. We also compared these results to two state-of-the-art methods for predicting protein-protein interactions: the PE score and the InSite probabilities. As we can see, by integrating multiple sources of data, we are able to improve the accuracy to 0.88 (area under the ROC curve) from 0.85 and 0.79 for PE score and InSite probabilities respectively. The PE score provides the strongest signal; using it alone or combining it with other subsets of data sources is able to predict complex-complex interaction with an accuracy that is slightly lower than our integrated model (Fig. 5.1).
CHAPTER 5. COMPLEX-COMPLEX INTERACTIONS

Figure 5.1: **Verification of complex-complex interactions.** Verification of our complex-complex interaction predictions relative to our reference set. Complex pairs in the hidden set in a ten-fold cross validation are ranked based on their predicted interaction probabilities. Blue, green, and red curves are for the three models we tried. Light blue and purple curves are for the predictions using only PE score or InSite probabilities respectively. Each point on the curve corresponds to a different threshold, giving rise to a different number of predicted interactions. The value on the x-axis is the number of pairs not in the reference set, but predicted to interact. The value on the y-axis is the number of reference interactions that are predicted to interact. The bars on the right bottom corner are the areas under the ROC curves. As we can see, our Naive Bayes model with EM achieves the highest accuracy. The prediction made by PE score alone is slightly worse than our integrated models.
5.7.2 Functional coherence of interacting complexes

We evaluate whether two interacting complexes are more likely to share the same functional category, which is not used in our training. We used functional categories from MIPS [97], which has 18 functional categories with average 684 proteins per category. A complex is assigned to a particular functional category if more than half of its components belong to the functional category. We only perform our evaluation on complex pairs where both complexes are assigned to some MIPS functional category.

We trained our model on the entire reference set of complex-complex interactions and perform the evaluation on the top 500 predicted pairs of interacting complexes. Among them, as we can see from Fig. 5.2, 59.2% consist of complexes that share the same MIPS functional category, compare to only 35.2% among the random complex pairs. Therefore, our predicted set of interacting complexes is functionally more coherent.

5.7.3 Accuracy of unified interaction network

We also apply our model to predict a unified network involving both proteins and complexes. In this case, we have both the interactions between two complexes and the interactions between a protein and a complex. As we can see from Fig. 5.3, by integrating multiple data sources, our Naive Bayes model with EM is able to achieve higher accuracy than using PE score alone. We generated predictions for all protein-complex pairs and complex-complex pairs by training on the entire reference set (see our supporting website for the complete list of the predictions [3]). Combined with high-quality protein-protein interaction predictions [133], we provide biologists with a unified interaction network involving both proteins and complexes.

5.7.4 Conditions when two complexes interact

Due to our use of condition-specific time-series expression data as a cue for determining interaction, our analysis can also provide hypotheses regarding the condition in which two complexes interact. Between each pair of interacting complexes, we find the two proteins with the highest correlation. We then list the condition when the
Figure 5.2: **Functional coherence of interacting complexes.** We verified if interacting complexes are more functionally coherent by checking whether they are more likely to be in the same MIPS functional category, which is not used in training. We only consider those interacting complexes if both of them are assigned to some MIPS category. We picked the top 500 predictions from our Naive Bayes model with EM, which integrates multiple sources of data. We also picked top 500 predictions by using PE score alone. We compared them to the complex pairs in our reference set and randomly picked pairs. The x-axis shows the proportion of interacting complexes that are assigned to the same MIPS category. As we can see, 59.2% of our predicted interacting complexes share the same MIPS category, while only 35.2% and 45.5% share the same category for random complex pairs and for those predicted by PE score alone, respectively. Therefore, our predicted set of interacting complexes is more functionally coherent. The reference complexes are the most coherent. That is expected because the functional classification of the complexes is sometimes derived from the same literature sources as the interactions between those complexes.
Figure 5.3: Verification of our unified interaction network. Verification of our predictions involving both protein-complex interactions and complex-complex interactions against the reference set. Complex pairs in the hidden set of a ten-fold cross validation are ranked based on their predicted interaction probabilities. Blue curve is for our Naive Bayes model with EM. Green curve is for the predictions using only PE score. Each point on the curve corresponds to a different threshold, giving rise to a different number of predicted interactions. The value on the x-axis is the number of pairs not in the reference set, but predicted to interact. The value on the y-axis is the number of reference interactions that are predicted to interact. The area under the blue curve is 0.82 and the area under the green curve is 0.73. Therefore, our data integration is able to achieve better accuracy than using a single data source alone.
two proteins are most correlated (see our supporting website [3]). This list provides the biologists with a clue about when the interaction occurs.

5.8 Discussion

With our high-quality set of predicted complexes from Chapter 4, we are able to take a higher-level perspective on the protein-protein interaction network, viewing it in terms of interactions between atomic units — whether individual proteins or stable complexes. There has been much work on predicting protein-protein interactions. However, these pairwise interactions are often induced by higher-level relationships: those within a complex and those between complexes. Interactions within a complex give rise to densely connected subgraphs in the interaction network; interactions between complexes can give rise to a network of interconnections involving different members of the two complexes. Viewing the network in terms of its atomic units can help clarify its structure and its basic properties. We therefore defined the novel problem of predicting interactions between complexes and other complexes or proteins, and constructed a new, high-accuracy method for making such predictions. The result of our analysis is a unified interaction network involving both proteins and complexes. We can now analyze the properties of this network, such as its connectivity and hierarchical structure, which better captures the true interactions underlying cellular processes.

Our work takes a step towards a more hierarchical view of the protein-protein interaction network, moving up from individual proteins to complexes as the basic interacting units. The next level of the hierarchy is the pathways that comprise cellular pathways. Although the notion of a ‘pathway’ is not as well-defined, it would nevertheless be very useful to reconstruct pathways that are comprised of interacting complexes and proteins. We can then move even higher in the cellular hierarchy, and study the interactions between pathways. This type of analysis will give us a unified perspective on the underlying hierarchical organization of the cell, and provide significant insight.
Chapter 6

Conclusions

In this final chapter, we summarize the contributions of this thesis and outline some future direction.

6.1 Summary

In this thesis, we try to gain understanding of the hierarchical structure of the protein dynamics by applying a diverse range of computational algorithms, adapted to the specific problem we want to solve and the characteristics of available data. At the lowest level, we try to predict binding sites of protein-protein interaction. We applied the framework of probabilistic graphical models to encode our prior knowledge about the relationship between different entities. Due to the lack of labeled data and direct evidence, we used unsupervised learning, which also takes into consideration the unlabeled data. At the middle level, we try to reconstruct a comprehensive set of stoichiometrically stable complexes. Here we have a reference set of complexes from small-scale experiments and large amount of direct evidence from high throughput experiments of relatively high quality. Therefore, we use supervised learning to combine the evidence and then tackle the complex reconstruction using a specifically designed clustering algorithm that allows overlap. In the end, at the highest level, we try to predict interactions between the stoichiometrically stable complexes we just constructed in the previous part. Here again we lack enough labeled data and direct
evidence so we used semi-supervised learning. Here we focus on feature construction to extract and aggregate information between two complexes. One useful feature is the protein-protein interactions we predicted in the first part. Therefore, the work of the previous two parts serves as the foundation for the last part, which deals with the highest level of interactions. The common theme across all parts of the thesis is the task of integrating heterogeneous types of noisy data.

Here is a list of our specific contributions:

**Biological:**

1. High quality and genome-wide predictions of protein-protein interactions and their binding sites.
2. A set of reference complexes that is merged from different sources with higher coverage.
3. High quality and genome-wide predictions of protein complexes.
4. A better way to process time-series expression data. Among many ways to process the data, this correlates the best with interactions between complexes.
5. High quality and genome-wide predictions of interactions between complexes and proteins.

All the above predictions can be downloaded from our website for further analysis by biologists.

**Computational:**

1. An algorithm that allows us to do fast MAP inference in MRF. [26]
2. An extension to the popular hierarchical agglomerative clustering (HAC) algorithm to allow overlaps (HACO) in the resulting clustering. Since HAC is shown to be useful in many tasks [34, 25, 26], we expect HACO to be also widely applicable.

All the above novel algorithms as well as the code that generated our biological predictions can be downloaded from our website. They are general-purpose and can be applied to a wide range of problems.
6.2 Future directions

6.2.1 Identifying pathways

In the last chapter, we found interacting complexes are more likely to share the same functional category. This is partly due to the fact that complexes interact with each other in the same pathway to achieve a certain biological task. Therefore, instead of predicting interactions between complexes, we can try to directly predict complexes that are within the same pathway. This is similar to what we did in Chapter 4 where we predicted complexes, instead of predicting interactions between proteins, which are partly a result of proteins belonging to the same complex.

To predict pathways, we may use the clustering algorithm that groups complexes into coherent sets, which are predicted to be our pathways. However unlike the task of identifying complexes, here we lack enough labeled data and high-quality direct measurement of complexes in the same pathway.

6.2.2 Different types of interactions

In this thesis, we try to predict interactions between biological entities: proteins and complexes. We view an interaction as a binary attribute: a pair of proteins or complexes either interact or not. In real biology, however, there are different types of interactions such as phosphorylation, methylation, or permanent binding. So instead of simply predicting the binary relationship, we can try to predict the exact type of interactions, which would provide biologists with much richer information. There are some preliminary study of the relationship between those different types of interactions. In particular, Zhang et al. [142] discovered motifs, which are different types of interactions arranged in certain patterns occur repeatedly in the interaction network. Those motifs suggest that there exists strong correlation between those types of interactions. Therefore, we may use a probabilistic graphical model to encode and learn those correlations. Once learned, we may use the model to make collective predictions on all types of interactions.
6.2.3 Interacting regions between complexes

In this thesis, we predicted whether two complexes interact or not. In the future, we can also try to predict which proteins on the complexes actually bind to each other, which, combined with our InSite model in Chapter 2, will give the exact location where a complex-complex interaction occurs. This will help us design drugs that specifically target the interaction sites and disable the complex-complex interaction.
Appendix A

Aggregate to create complex-level features

For each protein-level pairwise signal, we create the complex-level features using the following aggregation functions:

- Summation of all values.
- Maximum of all values.
- Average of all values.
- Minimum of all values.
- Number of values above a certain cutoff where the cutoff for different data sources are listed below.
- ‘number above cutoff’ divided by total number of values.
- ‘number above cutoff’ divided by square root of total number of values.
- Average of top three values.
- Decayed max: weighted average with the largest value has weight 1, second largest weight 1/2, and each subsequent one reducing the weight by half.
APPENDIX A. AGGREGATE TO CREATE COMPLEX-LEVEL FEATURES

- Decayed min: weighted average with the smallest value has weight 1, second smallest weight 1/2, and each subsequent one reducing the weight by half.

Here are the cutoffs used in the aggregating functions:

3 for direct PE score.

3 for indirect PE score.

0.5 for scaled PE score.

0.2 for PE score correlation.

7.5 for semantic distance of GO cell component.

7.5 for log size of the smallest possible GO group that could contain both proteins.

0.5 for trans-membrane proteins.

3 for the product of trans-membrane and direct PE score.

3 for the product of trans-membrane and indirect PE score.

0.5 for the product of trans-membrane and scaled PE score.

0.5 for expression correlation.

0.5 for Yeast two-hybrid.
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