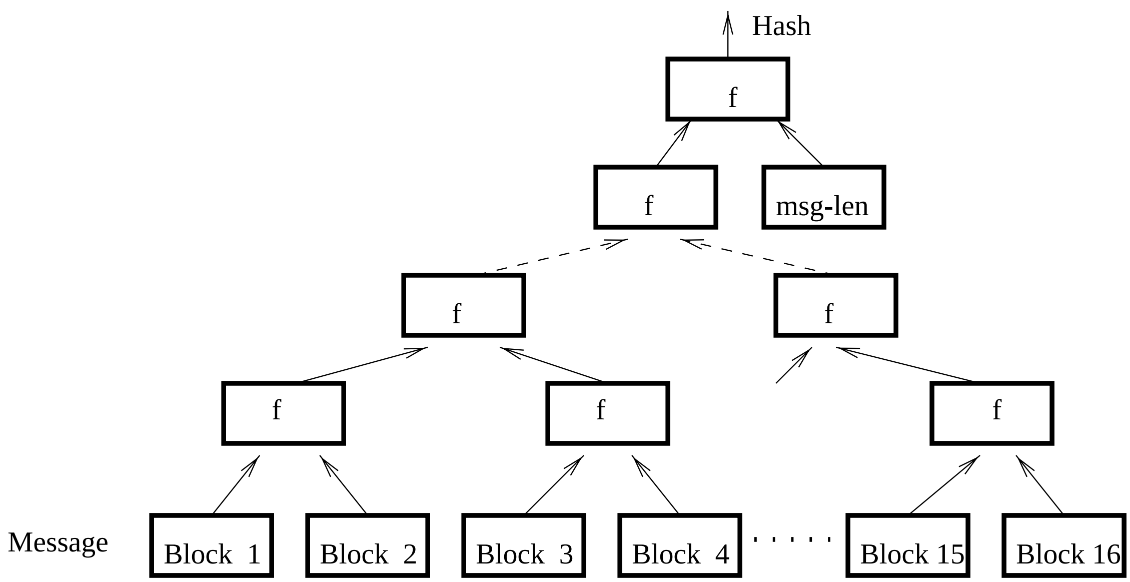
CS255: Cryptography and Computer Security **Winter 2000**

Assignment #3

Due: Monday, February 28th, 2000.

Problem 1 Merkle hash trees.

Merkle suggested a parallelizable method for constructing hash functions out of compression functions. Let f be a compression function that takes two 512 bit blocks and outputs one 512 bit block. To hash a message M one uses the following tree construction:



Prove that if one can find a collision for the resulting hash function then one can find collisions for the compression function.

Problem 2 In this problem we explore the different ways of constructing a MAC out of a non-keyed hash function. Let $h : \{0,1\}^* \to \{0,1\}^b$ be a hash function constructed by iterating a collision resistant compression function using the Merkle-Damgård construction.

1. Show that defining $MAC_k(M) = h(k \parallel M)$ results in an insecure MAC. That is, show that given a valid text/MAC pair (M, H) one can efficiently construct another valid text/MAC pair (M', H') without knowing the key k.
2. Recall that in the Merkle-Damgård iterated construction one uses a fixed Initial Value IV as the initial chaining variable. Show that setting the IV to be the secret key k results in an insecure MAC.

3. Consider the MAC defined by $MAC_k(M) = h(M \parallel k)$. Show that in expected time $O(2^{b/2})$ it is possible to construct two messages M and M' such that given $MAC_k(M)$ it is possible to construct $MAC_k(M')$ without knowing the key k.

4. Give a short high level argument to show why the envelope method for constructing a MAC out of a hash function produces a secure MAC.

Problem 3 Rabin suggested a signature scheme very similar to RSA signatures. In its simplest form, the public key is a product of two large primes $N = pq$ and the private key is p and q. The signature S of a message $M \in \mathbb{Z}_N$ is the square root of M modulo N. For simplicity, assume that the messages M being signed are always quadratic residues modulo N. To verify the signature, simply check that $S^2 = M \bmod N$. Note that we did not include any hashing of M prior to signing. Show that a chosen message attack on the scheme can result in a total break. More precisely, if an attacker can get Alice to sign messages chosen by the attacker then the attacker can factor N.

Hint: recall that a quadratic residue modulo $N = pq$ has four square roots in $\mathbb{Z}_N$. First show that there are two square roots of M that enable the attacker to factor N (use the fact that gcd's are easy to compute). Then show how using a chosen message attack the attacker can get a hold of such a pair of square roots. Note that proper hashing prior to signing prevents this attacks.

Problem 4 Suppose Alice and Bob share a secret key k. A simple proposal for a MAC algorithm on fixed length messages is as follows: given a message M do: (1) compute 128 different parity bits of M (i.e. compute the parity of 128 different subsets of the bits of M), and (2) DES encrypt the resulting 128-bit checksum using k. Naively, one could argue that without knowing k an attacker cannot compute the MAC of a message M. Show that this proposal is flawed. Note that the algorithm for computing the 128-bit checksum is public.

Hint: show that an attacker can carry out an existential forgery given one valid message/MAC pair. Use linear algebra modulo 2.

Extra Credit Recall that in the ElGamal signature scheme, a signature is of the form (a, b) where $b \in Z_q$ and a is an integer. In lecture, we glossed over the fact that a is required to be less than p. Show that without this restriction, one can forge signatures for any message.

Hint: ElGamal says to find (a, b) such that $y^a a^b = g^M$. First show how to find (b, c, d) such that $y^d c^b = g^M$.]