Robot Motion Planning, Chapter 1

Jean-Claude Latombe

notes by Urszula Chajewska
February 11, 1996

Aspects of Motion Planning

1. input: task description - geometry and physical properties of objects, spatial relations (can be provided by the user or obtained through sensing)
   〈 complete knowledge -- plans can be fully generated and then executed
   〈 incomplete knowledge -- some interweaving of planning and execution is necessary
   〈 well-bounded uncertainty -- robust plans anticipating various contingencies may be suitable
2. output: sequence of robot motions to be executed
3. motion coordination (mobile obstacles, other robots)

Basic Problem

<u>Assumptions</u>:

1. Robot is the only moving object in the workspace
2. Robot is a single rigid object
3. No kinematic constraints limit robot's motions (*free-flying object*)
4. Dynamic properties of the robot are ignored (no temporal issues)
5. Motions restricted to non-contact
6. Full knowledge of the workspace

Problem is reduced to geometrical path planning.

Configuration Space Formulation

<u>Goal</u>: transforming the problem of planning the motion of a dimensioned object into the problem of planning the motion of a point.

<u>Process</u> (see Figure 3 on page 11):

1. represent the robot as the coordinates of one of its vertices.
2. "grow" all the obstacles in the workspace by the shape of the robot.
3. find a path for the transformed problem

<u>Terminology</u>:

configuration q of A (A-the robot) is a specification of the position T and orientation Θ of the frame of A with respect to the workspace

configuration space of A is the space C of all the configurations of A

path of A from the configuration q_{init} to the configuration q_{goal} is a continuous map τ : [0, 1] à C with:

$\tau(0) = q_{init}$ and $\tau(1) = q_{goal}$

free configuration space C_{free} is a configuration space C minus the union of all configurations overlapping the "enlarged" obstacles

free configuration is any configuration in C_{free}

free path of A from the free configuration q_{init} to the free configuration q_{goal} is a continuous map

$\tau : [0, 1] \rightarrow C_{free}$ with: $\tau(0) = q_{init}$ and $\tau(1) = q_{goal}$

semi-free path is just like the free path except that the robot can touch obstacles on the way.

Given an initial and a goal configuration, the basic motion planning problem is to generate a free path between the two configuration, if they belong to the same connected component of C_{free} and to report failure otherwise.

Planning Approaches

1. Roadmap
   - visibility graph - see Figure 4 on page 13 (applies mainly to two-dimensional configuration spaces with a polygonal C-obstacle region) - undirected graph G where
     nodes: the initial and goal configuration and all the C-obstacle ("grown" obstacles) vertices.
     edges: all the straight line segments connecting two nodes that do not intersect the interior of the C-obstacle region
     G can be searched for the shortest semi-free path between initial and goal positions.
   - retraction
     see Figure 5 on page 14
2. Cell decomposition (see Figures 6.1 and 6.2 on pages 16 and 17)
   Exact method (union of cells = free space) - complete
   - decompose the free space into simple regions, called cells (such that a path between any two configurations in a cell can be easily generated)
   - create an undirected graph (*connectivity graph*) representing the adjacency relation between the cells
   - find the sequence of cells on the path from initial to goal cells (*channel*)
   - construct a free path

   Approximate method - produces cells of predefined shape (union of cells is strictly included in the free space) - easier to implement, but not necessarily complete ("resolution-complete" - the precision of the approximation can be made arbitrarily small).
3. Potential field

Metaphor: a robot is a particle moving under the influence of an artificial potential produced by the goal configuration (positive) and the obstacles (negative). The negated gradient of the total potential is treated as a force applied to the robot. At every configuration, the direction of this force is considered the most promising direction of motion.

Analogous to fastest descent optimization methods; can be very efficient; can get trapped into local minima.

Extensions of the Basic Problem

1. Multiple moving objects
   - Moving Obstacle - requires adding one dimension representing time to the robot's configuration space (*configuration-time space*). Path cannot go backward along the time axis - path planning method must be modified.
   - Multiple Robots - we can treat them as a) one multi-bodied robot (*composite configuration space, centralized planning*) or b) plan their motions independently

and consider the interactions among the paths in the second phase of planning (*decoupled planning*). a) is complete, but inefficient (the dimension of the composite configuration space is equal to the sum of the dimensions of the individual spaces); b) is efficient, but not complete (see Figure 9 on page 24).

- Articulated Robots (made of several moving rigid objects called *links* connected by *joints*) - the extension of the planning method usually straightforward, but the dimension of the configuration space grows with the number of joints.

2. Kinematic Constraints
   - holonomic constraints - example: joint in an articulated robot (constrains the motion on two links with respect to each other). Reduces the dimension of the configuration space. Doesn't raise new fundamental issues.
   - nonholonomic constraints - non-integrable equation involving the configuration parameters and their derivatives. Do not reduce the dimension of the space of possible differential motions. Much harder to deal with.
3. Uncertainty
   Can be related to: accuracy of motion, geometry of the robot and the obstacles, location of the obstacles, accuracy of sensors.
   Extreme case: need to explore the workspace (outside the realm of motion planning).
   Intermediate case: bounded uncertainty.
   - initial (I)and goal (G) regions, not configurations
   - the plan must guarantee to reach a configuration in G starting from any (unknown) configuration in I
   - plan must combine motion commands and sensor readings that interact at execution time
   - plan must terminate in the goal (reliable recognition of the goal is problematic)
   - can use *force-compliant* motion commands (robot may touch obstacle surfaces and slide on them)
4. Movable Objects - objects the robot can displace
   - use of intermediate goals (selecting locations for the objects to be moved to)
   - motion plan can be an alternate sequence of *transit* and *transfer* motions.
   - *transit* and *transfer* motions lie in different spaces (configuration space of the robot vs. configuration space of the union of the robot and the movable object moving with it). Problem: determining which spaces to use and when to change from one to another.
   - pushing vs. grasping

Computational Complexity

lower bound: PSPACE-hard for: 1. the basic problem with articulated robot with polyhedral links and polyhedral obstacles; 2. planar linkages; 3. multiple rectangles (multi-bodied robot) - also in PSPACE; 4. planar arm
upper bound: exponential in the number of the dimensions of the configuration space

Some simple settings can have polynomial time complexity usually due to small dimension of the configuration space and incomplete planning algorithms.

Reduction of Complexity

Approximation of motion planning problems into simpler ones.

1. Projection in Configuration Space

Replace the robot A by the surface or volume it sweeps out when it moves along r independent axes.
Example: replacing 2-dimensional object which can rotate with a disc.

2. Slicing in Configuration Space
Forbid the motion of A along some axes (possibly divide the problem into subproblems and forbid motion along some axes in some subproblems and along others in other subproblems). Not complete.
3. Simplification of the Shape of Objects
Approximate the shapes of both the robot and the obstacles by reducing the number of surfaces defining their boundary and/or the degree of the equations of these surfaces (usually by rectangloids or balls).
   - bounding approximation - new objects completely contain the original ones (sound, not complete)
   - bounded approximation - new objects are completely contained in the original ones (complete, not sound)
4. Focusing Attention on a Subset of the Workspace
Identify cluttered localities. Subproblems involving cluttered localities only require a restricted subset of the workspace to be considered. Hence, the number of constraints on the motions is considerably reduced. Cluttered localities are often stereotyped situations.

Relation to Other Problems

1. Interaction with Real-Time Motion Control - see Figure 17 on page 43
   - trajectory generation - deciding on the velocity profile along the path
   - trajectory tracking - computing the forces to be exerted by the actuators at each time
2. Interaction with Sensing
   - small uncertainty - anticipate all possible contingencies and generate plans beforehand
   - bigger uncertainty - a) potential field method on-line; b) tracking the boundaries of obstacles;
     c) combination of potential field method with cell decomposition (if some obstacles are unknown)
3. Interaction with Task-Level Planning

Goal is specified as a spatial arrangement of physical objects. Motion planner gets input from task planner (possibly gives feedback).

Literature Landmarks

Read this section for the historical account of the motion planning field over the past 20 years.